8 QUADRILATERALS

EXERCISE 8.1

Q.1. *The angles of a quadrilateral are in the ratio* 3 : 5 : 9 : 13. *Find all the angles of the quadrilateral.*

Sol. Suppose the measures of four angles are $3x$, $5x$, $9x$ and $13x$.

$\therefore$ $3x + 5x + 9x + 13x = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow$ $30x = 360°$

$\Rightarrow$ $x = \frac{360°}{30} = 12°$

$\Rightarrow$ $3x = 3 \times 12° = 36°$

$5x = 5 \times 12° = 60°$

$9x = 9 \times 12° = 108°$

$13x = 13 \times 12° = 156°$

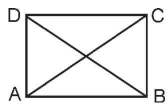
$\therefore$ the angles of the quadrilateral are **36°, 60°, 108° and 156° Ans.**

Q.2. *If the diagonals of a parallelogram are equal, then show that it is a rectangle.*

Sol. **Given :** ABCD is a parallelogram in which AC = BD.

To Prove : ABCD is a rectangle.

Proof : In ΔABC and ΔABD



AB = AB [Common]

BC = AD

[Opposite sides of a parallelogram]

AC = BD [Given]

$\therefore$ ΔABC ≅ ΔBAD [SSS congruence]

∠ABC = ∠BAD ...(i) [CPCT]

Since, ABCD is a parallelogram, thus,

∠ABC + ∠BAD = 180° ...(ii)

[Consecutive interior angles]

∠ABC + ∠ABC = 180°

$\therefore$ 2∠ABC = 180° [From (i) and (ii)]

$\Rightarrow$ ∠ABC = ∠BAD = 90°

This shows that ABCD is a parallelogram one of whose angle is 90°.

Hence, ABCD is a rectangle. **Proved.**

Q.3. *Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.*

Sol. **Given :** A quadrilateral ABCD, in which diagonals AC and BD bisect each other at right angles.

To Prove : ABCD is a rhombus.

Proof : In ΔAOB and ΔBOC

$AO = OC$

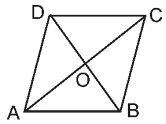
[Diagonals AC and BD bisect each other]

$\angle AOB = \angle COB$ [Each = 90°]

$BO = BO$ [Common]

$\therefore \Delta AOB \cong \Delta BOC$ [SAS congruence]

$AB = BC$ ...(i) [CPCT]



Since, ABCD is a quadrilateral in which

$AB = BC$ [From (i)]

Hence, ABCD is a rhombus.

[$\because$ if the diagonals of a quadrilateral bisect each other, then it is a parallelogram and opposite sides of a parallelogram are equal] **Proved.**

Q.4. *Show that the diagonals of a square are equal and bisect each other at right angles.*

Sol. **Given :** ABCD is a square in which AC and BD are diagonals.

To Prove : AC = BD and AC bisects BD at right angles, i.e. $AC \perp BD$. AO = OC, OB = OD

Proof : In ΔABC and ΔBAD,

$AB = AB$ [Common]

$BC = AD$ [Sides of a square]

$\angle ABC = \angle BAD = 90°$ [Angles of a square]

$\therefore \quad \Delta ABC \cong \Delta BAD$ [SAS congruence]

$\Rightarrow \quad AC = BD$ [CPCT]

Now in ΔAOB and ΔCOD,

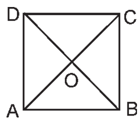
$AB = DC$ [Sides of a square]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate angles]

$\therefore \quad \Delta AOB \cong \Delta COD$ [AAS congruence]

$\angle AO = \angle OC$ [CPCT]



Similarly by taking ΔAOD and ΔBOC, we can show that OB = OD.

In ΔABC, $\angle BAC + \angle BCA = 90°$ [$\because \angle B = 90°$]

$\Rightarrow 2\angle BAC = 90°$ [$\angle BAC = \angle BCA$, as BC = AD]

$\Rightarrow \angle BCA = 45°$ or $\angle BCO = 45°$

Similarly $\angle CBO = 45°$

In ΔBCO.

$\angle BCO + \angle CBO + \angle BOC = 180°$

$\Rightarrow 90° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 90°$

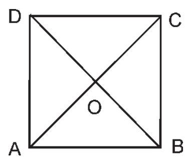
$\Rightarrow BO \perp OC \Rightarrow BO \perp AC$

Hence, AC = BD, $AC \perp BD$, AO = OC and OB = OD. **Proved.**

Q.5. *Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.*

Sol. **Given :** A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles,

To Prove : ABCD is a square.



Proof : Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

⇒ AB = BC = CD = DA [Sides of a rhombus]

In ΔABC and ΔBAD, we have

AB = AB [Common]

BC = AD [Sides of a rhombus]

AC = BD [Given]

∴ ΔABC ≅ ΔBAD [SSS congruence]

∴ ∠ABC = ∠BAD [CPCT]

But, ∠ABC + ∠BAD = 180° [Consecutive interior angles]

∠ABC = ∠BAD = 90°

∠A = ∠B = ∠C = ∠D = 90° [Opposite angles of a ||^{gm}]

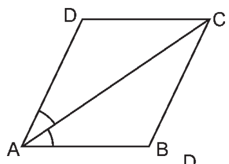
⇒ ABCD is a rhombus whose angles are of 90° each.

Hence, ABCD is a square. **Proved.**

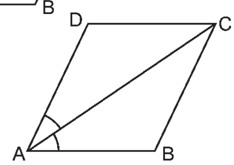
Q.6. *Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig.). Show that*

(i) it bisects ∠C also,

(ii) ABCD is a rhombus.



Given : A parallelogram ABCD, in which diagonal AC bisects ∠A, i.e., ∠DAC = ∠BAC.



To Prove : (i) Diagonal AC bisects ∠C i.e., ∠DCA = ∠BCA

(ii) ABCD is a rhomhus.

Proof : (i) ∠DAC = ∠BCA [Alternate angles]

∠BAC = ∠DCA [Alternate angles]

But, ∠DAC = ∠BAC [Given]

∴ ∠BCA = ∠DCA

Hence, AC bisects ∠DCB

Or, AC bisects ∠C **Proved.**

(ii) In ΔABC and ΔCDA

AC = AC [Common]

∠BAC = ∠DAC [Given]

and ∠BCA = ∠DAC [Proved above]

∴ ΔABC ≅ ΔADC [ASA congruence]

∴ BC = DC [CPCT]

But AB = DC [Given]

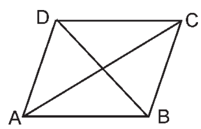
∴ AB = BC = DC = AD

Hence, ABCD is a rhombus **Proved.**

[∵ opposite angles are equal]

Q.7. *ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.*

Sol. **Given :** ABCD is a rhombus, i.e., AB = BC = CD = DA.



To Prove : ∠DAC = ∠BAC,

$\angle BCA = \angle DCA$

$\angle ADB = \angle CDB$, $\angle ABD = \angle CBD$

Proof : In ΔABC and ΔCDA, we have

$AB = AD$ [Sides of a rhombus]

$AC = AC$ [Common]

$BC = CD$ [Sides of a rhombus]

$\Delta ABC \cong \Delta ADC$ [SSS congruence]

So, $\left.\begin{aligned} \angle DAC &= \angle BAC \\ \angle BCA &= \angle DCA \end{aligned}\right\}$ [CPCT]

Similarly, $\angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$.

Hence, diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$. **Proved.**

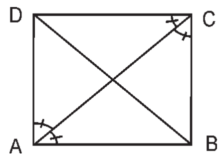
Q.8. *ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that :*

(i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. Given : ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$.

To Prove : (i) ABCD is a square.

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.



Proof : (i) In ΔABC and ΔADC, we have

$\angle BAC = \angle DAC$ [Given]

$\angle BCA = \angle DCA$ [Given]

$AC = AC$

$\therefore$ $\Delta ABC \cong \Delta ADC$ [ASA congruence]

$\therefore$ $AB = AD$ and $CB = CD$ [CPCT]

But, in a rectangle opposite sides are equal,

i.e., $AB = DC$ and $BC = AD$

$\therefore$ $AB = BC = CD = DA$

Hence, ABCD is a square **Proved.**

(ii) In ΔABD and ΔCDB, we have

$\left.\begin{aligned} AD &= CD \\ AB &= CD \end{aligned}\right\}$ [Sides of a square]

$BD = BD$ [Common]

$\therefore$ $\Delta ABD \cong \Delta CBD$ [SSS congruence]

So, $\left.\begin{aligned} \angle ABD &= \angle CBD \\ \angle ADB &= \angle CDB \end{aligned}\right\}$ [CPCT]

Hence, diagonal BD bisects $\angle B$ as well as $\angle D$ **Proved.**

Q.9. *In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig.). Show that :*

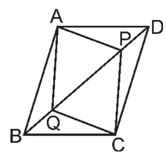
(i) $\Delta APD \cong \Delta CQB$

(ii) $AP = CQ$

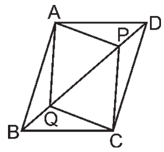
(iii) $\Delta AQB \cong \Delta CPD$

(iv) $AQ = CP$

(v) APCQ is a parallelogram



Sol. **Given :** ABCD is a parallelogram and P and Q are points on diagonal BD such that DP = BQ.



To Prove : (i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

Proof : (i) In ΔAPD and ΔCQB, we have

AD = BC [Opposite sides of a ||gm]

DP = BQ [Given]

$\angle ADP = \angle CBQ$ [Alternate angles]

$\therefore$ $\Delta APD \cong \Delta CQB$ [SAS congruence]

(ii) $\therefore$ AP = CQ [CPCT]

(iii) In ΔAQB and ΔCPD, we have

AB = CD [Opposite sides of a ||gm]

DP = BQ [Given]

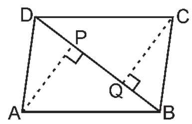
$\angle ABQ = \angle CDP$ [Alternate angles]

$\therefore$ $\Delta AQB \cong \Delta CPD$ [SAS congruence]

(iv) $\therefore$ AQ = CP [CPCT]

(v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram. **Proved.**

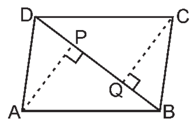
Q.10. *ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that*



(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Sol. **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.



To Prove : (i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

Proof : (i) In ΔAPB and ΔCQD, we have

$\angle ABP = \angle CDQ$ [Alternate angles]

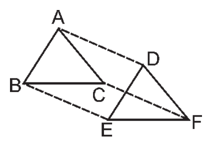
AB = CD [Opposite sides of a parallelogram]

$\angle APB = \angle CQD$ [Each = 90°]

$\therefore$ $\Delta APB \cong \Delta CQD$ [ASA congruence]

(ii) So, AP = CQ [CPCT] **Proved.**

Q.11. *In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.). Show that*



(i) quadrilateral ABED is a parallelogram

(ii) quadrilataeral BEFC is a parallelogram

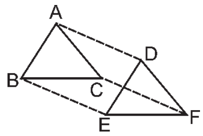
(iii) AD || CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) $\Delta ABC \equiv \Delta DEF$

Sol. **Given :** In DABC and DDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F.



To Prove : (i) ABED is a parallelogram

(ii) BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) ACFD is a parallelogram

(v) AC = DF

(vi) $\Delta ABC \cong \Delta DEF$

Proof : (i) In quadrilateral ABED, we have

AB = DE and AB || DE. [Given]

$\Rightarrow$ ABED is a parallelogram.

[One pair of opposite sides is parallel and equal]

(ii) In quadrilateral BEFC, we have

BC = EF and BC || EF [Given]

$\Rightarrow$ BEFC is a parallelogram.

[One pair of opposite sides is parallel and equal]

(iii) BE = CF and BE||BECF [BEFC is parallelogram]

AD = BE and AD||BE [ABED is a parallelogram]

$\Rightarrow$ AD = CF and AD||CF

(iv) ACFD is a parallelogram.

[One pair of opposite sides is parallel and equal]

(v) AC = DF [Opposite sides of parallelogram ACFD]

(vi) In ΔABC and ΔDEF, we have

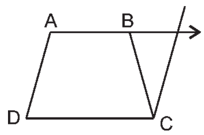
AB = DE [Given]

BC = EF [Given]

AC = DF [Proved above]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [SSS axiom] **Proved.**

Q.12. *ABCD is a trapezium in which AB || CD and AD = BC (see Fig.). Show that*



(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD

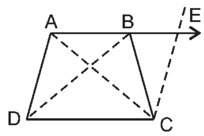
Sol. **Given :** In trapezium ABCD, AB || CD and AD = BC.

To Prove : (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD



Constructions : Join AC and BD. Extend AB and draw a line through C parallel to DA meeting AB produced at E.

Proof : (i) Since AB || DC

$\Rightarrow$ AE || DC ...(i)

and AD || CE ...(ii) [Construction]

$\Rightarrow$ ADCE is a parallelogram [Opposite pairs of sides are parallel

$\angle A + \angle E = 180°$ ...(iii) [Consecutive interior angles]

$\angle B + \angle CBE = 180°$ ...(iv) [Linear pair]

AD = CE ...(v) [Opposite sides of a $||^{gm}$]

AD = BC ...(vi) [Given]

$\Rightarrow$ BC = CE [From (v) and (vi)]

$\Rightarrow$ $\angle E = \angle CBE$ ...(vii) [Angles opposite to equal sides]

$\therefore \angle B + \angle E = 180°$ ...(viii) [From (iv) and (vii)

Now from (iii) and (viii) we have

$\angle A + \angle E = \angle B + \angle E$

$\Rightarrow$ $\angle A = \angle B$ **Proved.**

(ii) $\angle A + \angle D = 180°$
$\angle B + \angle C = 180°$ } [Consecutive interior angles]

$\Rightarrow \angle A + \angle D = \angle B + \angle C$ [$\because \angle A = \angle B$]

$\Rightarrow$ $\angle D = \angle C$

Or $\angle C = \angle D$ **Proved.**

(iii) In ΔABC and ΔBAD, we have

AD = BC [Given]

$\angle A = \angle B$ [Proved]

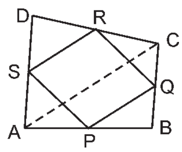
AB = CD [Common]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [ASA congruence]

(iv) diagonal AC = diagonal BD [CPCT] **Proved.**

EXERCISE 8.2

Q.1. *ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. (see Fig.). AC is a diagonal. Show that :*

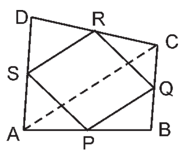


(i) SR || AC and SR = $\frac{1}{2}$*AC*

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Given : ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.



To Prove : (i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram

Proof : (i) In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC ...(1)

[Mid-point theorem]

In ΔADC, R is the mid-point of CD and S is the mid-point of AD

$\therefore$ SR || AC and SR = $\frac{1}{2}$AC ...(2)

[Mid-point theorem]

(ii) From (1) and (2), we get
PQ || SR and PQ = SR

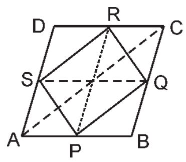
(iii) Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is equal and parallel.
$\therefore$ PQRS is a parallelogram. **Proved.**

Q.2. *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

Sol. **Given :** ABCD is a rhombus in which P, Q, R and S are mid points of sides AB, BC, CD and DA respectively :

To Prove : PQRS is a rectangle.

Construction : Join AC, PR and SQ.



Proof : In ΔABC

P is mid point of AB [Given]

Q is mid point of BC [Given]

$\Rightarrow$ PQ || AC and PQ = $\frac{1}{2}$AC ...(i) [Mid point theorem]

Similarly, in ΔDAC,

SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we have PQ||SR and PQ = SR

$\Rightarrow$ PQRS is a parallelogram

[One pair of opposite sides is parallel and equal]

Since ABQS is a parallelogram

$\Rightarrow$ AB = SQ [Opposite sides of a || gm]

Similarly, since PBCR is a parallelogram.

$\Rightarrow$ BC = PR

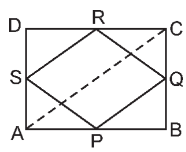
Thus, SQ = PR [AB = BC]

Since SQ and PR are diagonals of parallelogram PQRS, which are equal.

$\Rightarrow$ PQRS is a rectangle. **Proved.**

Q.3. *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilataral PQRS is a rhombus.*

Sol. **Given :** A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively, PQ, QR, RS and SP are joined.



To Prove : PQRS is a rhombus.

Construction : Join AC

Proof : In ΔABC, P and Q are the mid-points of the sides AB and BC.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC ...(i) [Mid point theorem]

Similarly, in ΔADC,

SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we get

PQ || SR and PQ = SR ...(iii)

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From (iii)]

$\therefore$PQRS is a parallelogram.

Now AD = BC ...(iv)

[Opposite sides of a rectangle ABCD]

$\therefore \quad \frac{1}{2}AD = \frac{1}{2}BC$

$\Rightarrow \quad$ AS = BQ

In ΔAPS and ΔBPQ

AP = BP [$\because$ P is the mid-point of AB]

AS = BQ [Proved above]

$\angle PAS = \angle PBQ$ [Each = 90°]

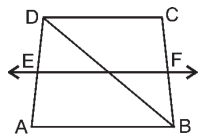
$\Delta APS \cong \Delta BPQ$ [SAS axiom]

$\therefore$ PS = PQ ...(v)

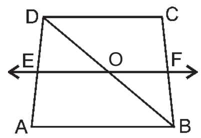
From (iii) and (v), we have

PQRS is a rhombus **Proved.**

Q.4. *ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig.). Show that F is the mid-point of BC.*



Sol. Given : A trapezium ABCD with AB || DC, E is the mid-point of AD and EF || AB.



To Prove : F is the mid-point of BC.

Proof : AB || DC and EF || AB

$\Rightarrow$ AB, EF and DC are parallel.

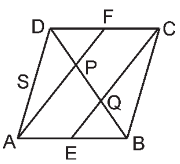
Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.

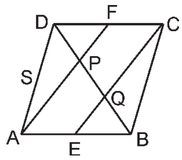
$\therefore$ Intercepts made by those parallel lines on transversal BC are also equal.

i.e., BF = FC

$\Rightarrow$ F is the mid-point of BC.

Q.5. *In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.). Show that the line segments AF and EC trisect the diagonal BD.*





Sol. Given : A parallelogram ABCD, in which E and F are mid-points of sides AB and DC respectively.

To Prove : DP = PQ = QB

Proof : Since E and F are mid-points of AB and DC respectively.

$\Rightarrow$ AE = $\frac{1}{2}$AB and CF = $\frac{1}{2}$DC ...(i)

But, AB = DC and AB || DC ...(ii)

[Opposite sides of a parallelogram]

$\therefore$ AE = CF and AE || CF.

$\Rightarrow$ AECF is a parallelogram.

[One pair of opposite sides is parallel and equal]

In ΔBAP,

E is the mid-point of AB

EQ || AP

$\Rightarrow$ Q is mid-point of PB [Converse of mid-point theorem]

$\Rightarrow$ PQ = QB ...(iii)

Similarly, in ΔDQC,

P is the mid-point of DQ

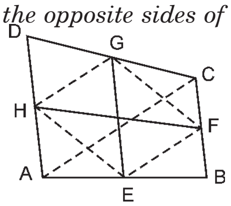
DP = PQ ...(iv)

From (iii) and (iv), we have

DP = PQ = QB

or line segments AF and EC trisect the diagonal BD. **Proved.**

Q.6. *Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.*



Sol. Given : ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.

To Prove : EG and FH bisect each other.

Construction : Join EF, FG, GH, HE and AC.

Proof : In ΔABC, E and F are mid-points of AB and BC respectively.

$\therefore$ EF = $\frac{1}{2}$AC and EF || AC ...(i)

In ΔADC, H and G are mid-points of AD and CD respectively.

$\therefore$ HG = $\frac{1}{2}$AC and HG || AC ...(ii)

From (i) and (ii), we get

EF = HG and EF || HG

$\therefore$ EFGH is a parallelogram.

[$\because$ a quadrilateral is a parallelogram if its one pair of opposite sides is equal and parallel]

Now, EG and FH are diagonals of the parallelogram EFGH.

$\therefore$ EG and FH bisect each other.

[Diagonal of a parallelogram bisect each other] **Proved.**

Q.7. *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

(i) D is the mid-point of AC.

(ii) MD ⊥ AC

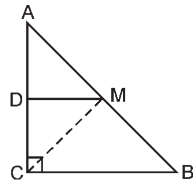
(iii) $CM = MA = \frac{1}{2}AB$

Sol. Given : A triangle ABC, in which ∠C = 90° and M is the mid-point of AB and BC || DM.

To Prove : (i) D is the mid-point of AC [Given]

(ii) DM ⊥ BC

(iii) $CM = MA = \frac{1}{2}AB$



Construction : Join CM.

Proof : (i) In ΔABC,

M is the mid-point of AB. [Given]

BC || DM [Given]

D is the mid-point of AC

[Converse of mid-point theorem] **Proved.**

(ii) ∠ADM = ∠ACB [∵ Coresponding angles]

But ∠ACB = 90° [Given]

∴ ∠ADM = 90°

But ∠ADM + ∠CDM = 180° [Linear pair]

∴ ∠CDM = 90°

Hence, MD ⊥ AC **Proved.**

(iii) AD = DC ...(1) [∵ D is the mid-point of AC]

Now, in ΔADM and ΔCMD, we have

∠ADM = ∠CDM [Each = 90°]

AD = DC [From (1)]

DM = DM [Common]

∴ ΔADM ≅ ΔCMD [SAS congruence]

⇒ CM = MA ...(2) [CPCT]

Since M is mid-point of AB,

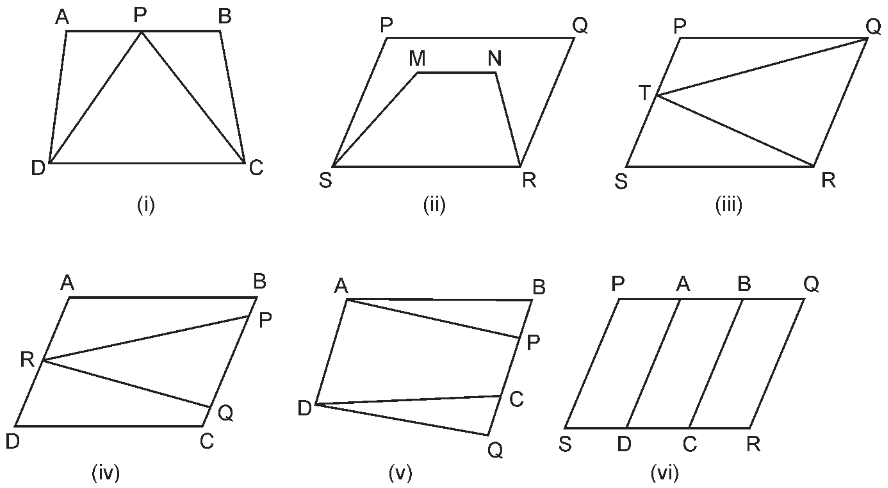
∴ $MA = \frac{1}{2}AB$ ...(3)

Hence, $CM = MA = \frac{1}{2}AB$ **Proved.** [From (2) and (3)]

9 AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.1

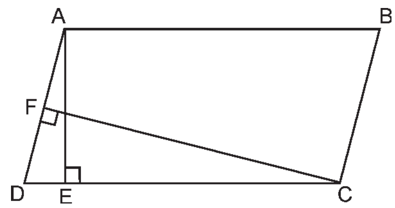
Q.1. *Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.*



Sol. (i) Base DC, parallels DC and AB
(iii) Base QR, parallels QR and PS
(v) Base AD, parallels AD and BQ.

EXERCISE 9.2

Q.1. *In the figure, ABCD is a paralle-logram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.*



Sol. Area of parallelogram ABCD

$= AB \times AE$

$= 16 \times 8 \text{ cm}^2 = 128 \text{ cm}^2$

Also, area of parallelogram ABCD

$= AD \times FC = (AD \times 10) \text{ cm}^2$

$\therefore \quad AD \times 10 = 128$

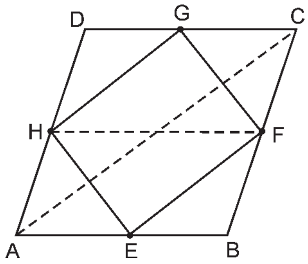
$\Rightarrow \quad AD = \frac{128}{10}$ = **12.8 cm Ans.**

Q.2. *If E, F, G, and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) =* $\frac{1}{2}$ *ar (ABCD).*

Sol. **Given :** A parallelogram ABCD · E, F, G, H are mid-points of sides AB, BC, CD, DA respectively

To Porve : ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Construction : Join AC and HF.



Proof : In ΔABC,
E is the mid-point of AB.
F is the mid-point of BC.

$\Rightarrow$ EF is parallel to AC and EF = $\frac{1}{2}$ AC ... (i)

Similarly, in ΔADC, we can show that

HG $\parallel$ AC and HG = $\frac{1}{2}$ AC ... (ii)

From (i) and (ii)
EF $\parallel$ HG and EF = HG
$\therefore$ EFGH is a parallelogram. [One pour of opposite sides is equal and parallel]

In quadrilateral ABFH, we have

HA = FB and HA $\parallel$ FB [AD = BC $\Rightarrow \frac{1}{2}$AD = $\frac{1}{2}$BC $\Rightarrow$ HA = FB]

$\therefore$ ABFH is a parallelogram. [One pair of opposite sides is equal and parallel]

Now, triangle HEF and parallelogram HABF are on the same base HF and between the same parallels HF and AB.

$\therefore$ Area of $\Delta HEF = \frac{1}{2}$ area of HABF ... (iii)

Similarly, area of $\Delta HGF = \frac{1}{2}$ area of HFCD ... (iv)

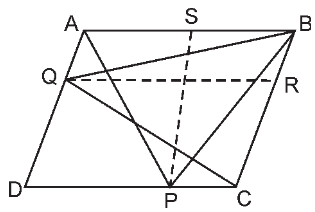
Adding (iii) and (iv),
Area of ΔHEF + area of ΔHGF

$= \frac{1}{2}$ (area of HABF + area of HFCD)

$\Rightarrow$ ar (EFGH) $= \frac{1}{2}$ ar (ABCD) **Proved.**

Q.3. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).*

Sol. Given : A parallelogram ABCD. P and Q are any points on DC and AD respectively.



To prove : ar (APB) = ar (BQC)

Construction : Draw PS $\parallel$ AD and QR $\parallel$ AB.

Proof : In parallelogram ABRQ, BQ is the diagonal.

$\therefore$ area of $\Delta BQR = \frac{1}{2}$ area of ABRQ ... (i)

In parallelogram CDQR, CQ is a diagonal.

$\therefore$ area of $\Delta RQC = \dfrac{1}{2}$ area of CDQR ... (ii)

Adding (i) and (ii), we have

area of ΔBQR + area of ΔRQC

$= \dfrac{1}{2}$ [area of ABRQ + area of CDQR]

$\Rightarrow$ area of $\Delta BQC = \dfrac{1}{2}$ area of ABCD ... (iii)

Again, in parallelogram DPSA, AP is a diagonal.

$\therefore$ area of $\Delta ASP = \dfrac{1}{2}$ area of DPSA ... (iv)

In parallelogram BCPS, PB is a diagonal.

$\therefore$ area of $\Delta BPS = \dfrac{1}{2}$ area of BCPS ... (v)

Adding (iv) and (v)

area of ΔASP + area of $\Delta BPS = \dfrac{1}{2}$(area of DPSA + area of BCPS)

$\Rightarrow$ area of $\Delta APB = \dfrac{1}{2}$ (area of ABCD) ... (vi)

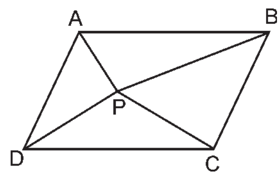
From (iii) and (vi), we have

area of ΔAPB = area of ΔBQC. **Proved.**

Q.4. *In the figure, P is a point in the interior of a parallelogram ABCD. Show that*

(i) ar (APB) + ar (PCD) = $\dfrac{1}{2}$ *ar (ABCD)*

(ii) ar (APD) + ar (PBC) = ar(APB) + ar (PCD)



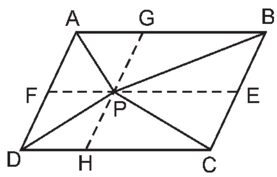
Sol. **Given :** A parallelogram ABCD. P is a point inside it.

To prove : (i) ar (APB) + ar(PCD)

$= \dfrac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC)

= ar (APB) + ar (PCD)



Construction : Draw EF through P parallel to AB, and GH through P parallel to AD.

Proof : In parallelogram FPGA, AP is a diagonal,

$\therefore$ area of ΔAPG = area of ΔAPF ... (i)

In parallelogram BGPE, PB is a diagonal,

$\therefore$ area of ΔBPG = area of ΔEPB ... (ii)

In parallelogram DHPF, DP is a diagonal,

∴ area of ΔDPH = area of ΔDPF ... (iii)

In parallelogram HCEP, CP is a diagonal,

∴ area of ΔCPH = area of ΔCPE ... (iv)

Adding (i), (ii), (iii) and (iv)

area of ΔAPG + area of ΔBPG + area of ΔDPH + area of ΔCPH

= area of ΔAPF + area of ΔEPB + area of ΔDPF + area ΔCPE

⇒ [area of ΔAPG + area of ΔBPG] + [area of ΔDPH + area of ΔCPH]

= [area of ΔAPF + area of ΔDPF] + [area of ΔEPB + area of ΔCPE]

⇒ area of ΔAPB + area of ΔCPD = area of ΔAPD + area of ΔBPC ... (v)

But area of parallelogram ABCD

= area of ΔAPB + area of ΔCPD + area of ΔAPD + area of ΔBPC ... (vi)

From (v) and (vi)

area of ΔAPB + area of ΔPCD = $\frac{1}{2}$ area of ABCD

or, ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD) **Proved.**

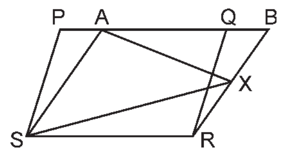
(ii) From (v),

⇒ ar (APD) + ar (PBC) = ar (APB) + ar (CPD) **Proved.**

Q.5. *In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that*

(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ *ar (PQRS)*



Sol. Given : PQRS and ABRS are parallelograms and X is any point on side BR.

To prove : (i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Proof : (i) In ΔASP and BRQ, we have

∠SPA = ∠RQB [Corresponding angles] ...(1)

∠PAS = ∠QBR [Corresponding angles] ...(2)

∴ ∠PSA = ∠QRB [Angle sum property of a triangle] ...(3)

Also, PS = QR [Opposite sides of the parallelogram PQRS] ...(4)

So, ΔASP ≅ ΔBRQ [ASA axiom, using (1), (3) and (4)]

Therefore, area of ΔPSA = area of ΔQRB

[Congruent figures have equal areas] ...(5)

Now, ar (PQRS) = ar (PSA) + ar (ASRQ]

= ar (QRB) + ar (ASRQ]

= ar (ABRS)

So, ar (PQRS) = ar (ABRS) **Proved.**

(ii) Now, ΔAXS and ||gm ABRS are on the same base AS and between same parallels AS and BR

$\therefore$ area of $\Delta AXS = \frac{1}{2}$ area of ABRS

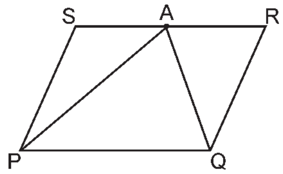
$\Rightarrow$ area of $\Delta AXS = \frac{1}{2}$ area of PQRS [$\because$ ar (PQRS) = ar (ABRS]

$\Rightarrow$ ar of (AXS) = $\frac{1}{2}$ ar of (PQRS) **Proved.**

Q.6. *A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?*

Sol. The field is divided in three triangles.

Since triangle APQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.



$\therefore$ ar (APQ) = $\frac{1}{2}$ ar (PQRS)

$\Rightarrow$ 2ar (APQ) = ar(PQRS)

But ar (PQRS) = ar(APQ) + ar (PSA) + ar (ARQ)

$\Rightarrow$ 2 ar (APQ) = ar(APQ) + ar(PSA) + ar (ARQ)

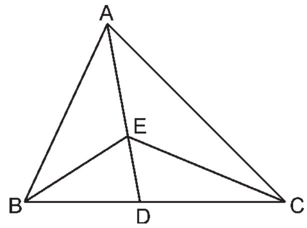
$\Rightarrow$ ar (APQ) = ar(PSA) + ar(ARQ)

Hence, area of ΔAPQ = area of ΔPSA + area of ΔARQ.

To sow wheat and pulses in equal portions of the field separately, farmer sow wheat in ΔAPQ and pulses in other two triangles or pulses in ΔAPQ and wheat in other two triangles. **Ans.**

EXERCISE 9.3

Q.1. *In the figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).*



Sol. **Given :** A triangle ABC, whose one median is AD. E is a point on AD.

To Prove : ar (ABE) = ar (ACE)

Proof : Area of ΔABD = Area of ΔACD ... (i)

[Median divides the triangle into two equal parts]

Again, in ΔEBC, ED is the median, therefore,

Area of ΔEBD = area of ΔECD .... (ii)

[Median divides the triangle into two equal parts]

Subtracting (ii) from (i), we have

area of ΔABD – area of ΔEBD = area of ΔACD – area of ΔECD

$\Rightarrow$ area of ΔABE = area of ΔACE

$\Rightarrow$ ar (ABE) = ar (ACE) **Proved.**

Q.2. *In a triangle ABC, E is the mid-point on median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).*

Sol. **Given :** A triangle ABC, in which E is the mid-point of median AD.

To Prove : ar(BED) = $\frac{1}{4}$ ar (ABC)

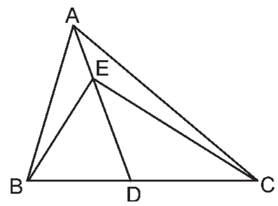
Proof : In ΔABC, AD is the median.

$\therefore$ area of ΔABD = area of ΔADC ... (i)

[Median divides the triangle into two equal parts]

Again, in ΔADB, BE is a median.

$\therefore$ area of ΔABE = area of ΔBDE ... (ii)



From (i), we have

area of $\Delta ABD = \frac{1}{2}$ area of ΔABC ... (iii)

From (ii), we have

area of $\Delta BED = \frac{1}{2}$ area of ΔABD ... (iv)

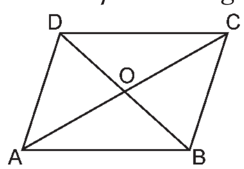
From (iii) and (iv), we have

area of $\Delta BED = \frac{1}{2} \times \frac{1}{2}$ area of ΔABC

$\Rightarrow$ area of $\Delta BED = \frac{1}{4}$ area of ΔABC

$\Rightarrow$ ar (BED) = $\frac{1}{4}$ ar(ABC) **Proved.**

Q.3. *Show that the diagonals of a parallelogram divide it into four triangles of equal area.*



Sol. Given : A parallelogram ABCD.

To Prove : area of ΔAOB = area of ΔBOC
= area of ΔCOD = area of ΔAOD

Proof : AO = OC and BO = OD

[Diagonals of a parallelogram bisect each other]

In ΔABC, O is mid-point of AC, therefore, BO is a median.

$\therefore$ area of ΔAOB = area of ΔBOC ... (i)

[Median of a triangle divides it into two equal parts]

Similarly, in ΔCBD, O is mid-point of DB, therefore, OC is a median.

$\therefore$ area of ΔBOC = area of ΔDOC ... (ii)

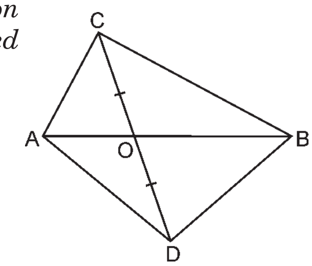
Similarly, in ΔADC, O is mid-point of AC, therefore, DO is a median.

$\therefore$ area of ΔCOD = area of ΔDOA ... (iii)

From (i), (ii) and (iii), we have

area of ΔAOB = area of ΔBOC = area of ΔDOC = area of ΔAOD **Proved.**

Q.4. *In the figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bsisected by AB at O, show that ar (ABC) = ar (ABD).*



Sol. Given : ABC and ABD are two triangles on the same base AB and line segment CD is bisected by AB at O.

To Prove : ar (ABC) = ar(ABD)

Proof : In ΔACD, we have

CO = OD [Given]

$\therefore$ AO is a median.

$\therefore$ area of ΔAOC = area of ΔAOD ... (i)

[Median of a triangle divides it into two equal parts]

Similarly, in ΔBCD, OB is median

$\therefore$ area of ΔBOC = area of ΔBOD ... (ii)

Adding (i) and (ii), we get

area of ΔAOC + area of ΔBOC = area of ΔAOD + area of ΔBOD

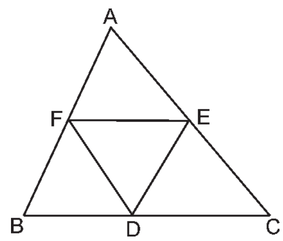
$\Rightarrow$ area of ΔABC = area of ΔABD

$\Rightarrow$ ar(ABC) = ar (ABD) **Proved.**

Q.5. *D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that*

(i) BDEF is a parallelogram. *(ii) ar(DEF) = $\frac{1}{4}$ ar(ABC)*

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)



Sol. Given : D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC.

To Prove : (i) BDEF is a parallelogram.

(ii) ar(DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar(BDEF) = $\frac{1}{2}$ ar (ABC)

Proof : (i) In ΔABC, E is the mid-point of AC and F is the mid-point of AB.

$\therefore$ EF || BC or EF || BD

Similarly, DE || BF.

$\therefore$ BDEF is a parallelogram ... (1)

(ii) Since DF is a diagonal of parallelogram BDEF.

Therefore, area of ΔBDF = area of ΔDEF ... (2)

Similarly, area of ΔAFE = area of ΔDEF ... (3)

and area of ΔCDE = area of ΔDEF ... (4)

From (2), (3) and (4), we have

area of ΔBDF = area of ΔAFE = area of ΔCDE = area of ΔDEF ... (5)

Again ΔABC is divided into four non-overlapping triangles BDF, AFE, CDE and DEF.

$\therefore$ area of ΔABC = area of ΔBDF + area of ΔAFE + area of ΔCDE + area of ΔDEF

= 4 area of ΔDEF ... (6) [Using (5)]

$\Rightarrow$ area of ΔDEF = $\frac{1}{4}$ area of ΔABC

$\Rightarrow$ ar (DEF) = $\frac{1}{4}$ ar (ABC) **Proved.**

(iii) Now, area of parallelogram BDEF = area of ΔBDF + area of ΔDEF

= 2 area of ΔDEF

= $2 \cdot \frac{1}{4}$ area of ΔABC

= $\frac{1}{2}$ area of ΔABC

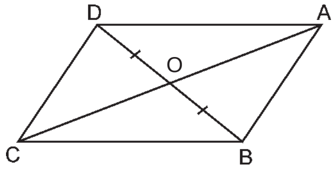
Hence, ar (BDEF) = $\frac{1}{2}$ ar (ABC) **Proved.**

Q.6. *In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that :*

(i) ar (DOC) = ar(AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CD or ABCD is a parallelogram.



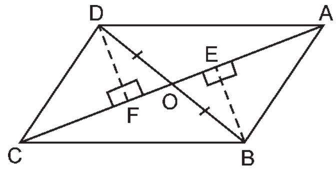
Sol. Given : Diagonal AC and BD of quadrilateral ABCD intersect at O such that OB = OD and AB = CD.

To Prove : (i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

Construction : Draw perpendiculars DF and BE on AC.



Proof : (i) area of ΔDCO = $\frac{1}{2}$ CO × DF ... (1)

area of ΔABO = $\frac{1}{2}$ AO × BE ... (2)

In ΔBEO and ΔDFO, we have

BO = DO [Given]

∠BOE = ∠DOF [Vertically opposite angles]

∠BEO = ∠DFO [Each = 90°]

⇒ ΔBOE ≅ ΔBOF [SAS congruence]

⇒ BE = DF [CPCI] ... (3)

OE = OF [CPCT] ... (4)

In ΔABE and ΔCDF, we have,

AB = CD [Given]

BE = DF [Proved above]

∠AEB = ∠CFD [Each = 90°]

∴ ΔABE ≅ CDF [RHS congruence]

⇒ AE = CF [CPCT] ... (5)

From (4) and (5), we have

OE + AE = OF + CF

⇒ AO = CO ... (6)

Hence, ar (DOC) = ar (AOB). [From (1), (2), (3) and (6)] **Proved.**

(ii) In quadrilateral ABCD, AC and BD are its diagonals, which intersect at 0.

Also, BO = OD [Given]

AO = OC [Proved above]

⇒ ABCD is a parallelogram [Diagonals of a quadrilateral bisect each other]

⇒ BC||AD.

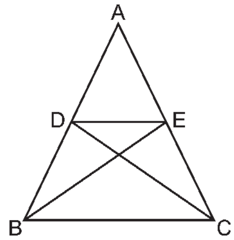
So, ar(DCB) = ar(DCB) **Proved.**

(iii) In (ii), we have proved that ABCD is a parallelogram.

Hence, ABCD is a parallelograms **Proved.**

Q.7. *D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE || BC.*

Sol. **Given :** D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC)



To Prove : DE $\parallel$ BC

Proof : ar (DBC) = ar (EBC) [Given]

Also, triangles DBC and EBC are on the same base BC.

$\therefore$ they are between the same parallels

i.e., DE $\parallel$ BC **Proved.**

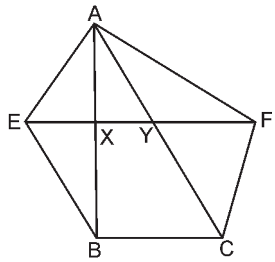
[$\because$ triangles on the same base and between the same parallels are equal in area]

Q.8. *XY is a line parallel to side BC of a triangle ABC. If BE $\parallel$ AC and CF $\parallel$ AB meet XY at E and F respectively, show that*

ar (ABE) = ar (ACF)

Sol. **Given :** XY is a line parallel to side BC of a ΔABC.

BE $\parallel$ AC and CF $\parallel$ AB



To Prove : ar (ABE) = ar (ACF)

Proof : ΔABE and parallelogram BCYE are on the same base BC and between the same parallels BE and AC.

$\therefore$ ar (ABE) = $\frac{1}{2}$ ar (BCYE) ... (i)

Similarly,

ar (ACF) = $\frac{1}{2}$ ar (BCFX) ... (ii)

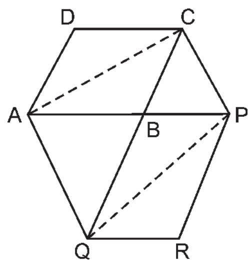
But parallelogram BCYE and BCFX are on the same base BC and between the same parallels BC and EF.

$\therefore$ ar (BCYE) = ar (BCFX) ... (iii)

From (i), (ii) and (iii), we get

ar (ABE) = ar (ACF) **Proved.**

Q.9. *The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see figure,). Show that ar (ABCD) = ar (PBQR).*



Sol. **Given :** ABCD is a parallelogram.

CP $\parallel$ AQ, BP $\parallel$ QR, BQ $\parallel$ PR

To Prove : ar (ABCD) = ar (PBQR)

Construction : Join AC and PQ.

Proof : AC is a diagonal of parallelogram ABCD.

$\therefore$ area of ΔABC = $\frac{1}{2}$ area of ABCD ... (i)

[A diagonal divides the parallelogram into two parts of equal area]

Similarly, area of $\Delta PBQ = \frac{1}{2}$ area of PBQR ... (ii)

Now, triangles AQC and AQP are on the same base AQ and between the same parallels AQ and CP.

$\therefore$ area of ΔAQC = area of ΔAQP ... (iii)

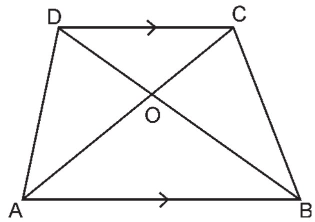
Subtracting area of ΔAQB from both sides of (iii),

area of ΔAQC – area of ΔAQB = area of ΔAQP – area of ΔAQB

$\Rightarrow$ area of ΔABC = area of ΔPBQ ... (iv)

$\Rightarrow \frac{1}{2}$ area of ABCD = $\frac{1}{2}$ area of PBQR [From (i) and (ii)]

$\Rightarrow$ area of ABCD = area of PBQR **Proved.**



Q.10. *Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).*

Sol. **Given :** Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

To Prove : ar (AOD) = ar (BOC)

Proof : Triangles ABC and BAD are on the same base AB and between the same parallels AB and DC.

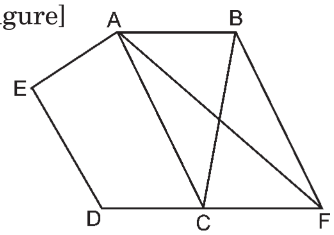
$\therefore$ area of ΔABC = area of ΔBAD

$\Rightarrow$ area of ΔABC – area of ΔAOB = area of ΔABD – area of ΔAOB

[subtracting area of ΔAOB from both sides]

$\Rightarrow$ area of ΔBOC = area of ΔAOD [From figure]

Hence, ar (BOC) = ar (AOD) **Proved.**



Q.11. *In the Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that*

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Sol. **Given :** ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

To Prove : (i) ar(ACB) = ar (ACF)

(ii) ar(AEDF) = ar (ABCDE)

Proof : (i) ΔACB and ΔACF lie on the same base AC and between the same parallels AC and BF.

Therefore, ar (ACB) = ar (ACF) **Proved.**

(ii) So, ar (ACB) + ar (ACDE) = ar (ACF) + ar (ACDE)

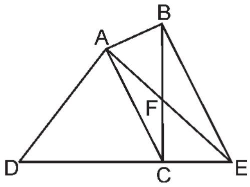
[Adding same areas on both sides]

$\Rightarrow$ ar (ABCDE) = ar(AEDF) **Proved.**

Q.12. *A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.*

Sol. ABCD is the plot of land in the shape of a quadrilateral. From B draw BE | |AC to meet DC produced at E.

To Prove : ar (ABCD) = ar (ADE)



Proof : ΔBAC and ΔEAC lie on the same base AC and between the same parallels AC and BE.

Therefore, ar (BAC) = ar (EAC)

So, ar (BAC) + ar (ADC) = ar (EAC) + ar (ADC)

[Adding same area on both sides]

or, ar (ABCD) = ar (ADE)

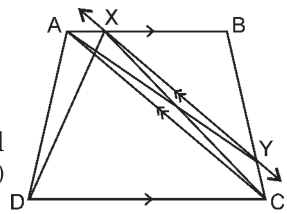
Hence, the gram Panchayat took over ABD and gave ΔCEF.

Q.13. *ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).*

Sol. Given : ABCD is a trapezium with AB || DC.

AC || XY.

To Prove : ar(ΔADX) = ar(ΔACY).



Construction : Join XC

Proof : Since AB || DC ∴ AX || DC

⇒ ar (ADX) = ar (AXC) ... (i)

(Having same base AX and between same parallels)

Since AC || XY

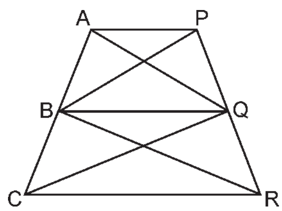
⇒ ar (AXC) = ar (ACY) ... (ii)

(Having same base AC and between same parallels)

⇒ ar (ADX) = ar (ACY) [From (i), (ii)] **Proved.**

Q.14. *In the figure, AP || BQ || CR. Prove that ar(AQC) = ar (PBR).*

Sol. Given : In figure, AP || BQ || CR.



To Prove : ar (AQC) = ar (PBR)

Proof : Triangles ABQ and PBQ are on the same base BQ and between the same parallels AP and BQ.

∴ ar(ABQ) = ar(PBQ) ... (1)

[Triangles on the same base and between the same parallels are equal in area]

Similarly triangle BQC and BQR on the same base BQ and between the same parallels BQ and CR

∴ ar(BQC) = ar (BQR) ... (2) [Same reason]

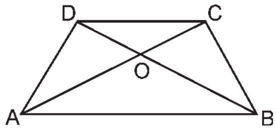
Adding (1) and (2), we get

ar (ABQ) + ar (BQC) = ar (PBQ) + ar (BQR)

⇒ ar(AQC) = ar (PBR). **Proved.**

Q.15. *Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.*

Sol. Given : Diagonals AC and BD of a quadrilateral ABCD intersect at O, such that ar (AOD) = ar (BOC)



To Prove : ABCD is a trapezium.

Proof : ar (ΔAOD) = ar(ΔBOC)

⇒ ar(AOD) + ar(BOA) = ar (BOC) + ar (BOA)

⇒ ar(ABD) = ar(ABC)

But, triangle ABD and ABC are on the same base AB and have equal area.

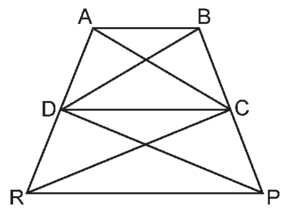
∴ they are between the same parallels,

i.e., AB || DC

Hence, ABCD is a trapezium. [∵ A pair of opposite sides is parallel]

Proved.

Q.16. *In the figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.*



Sol. Given : ar(DRC) = ar (DPC) and ar (BDP) = ar (ARC)

To Prove : ABCD and DCPR are trapeziums.

Proof : ar (BDP) = ar (ARC)

⇒ ar (DPC) + ar (BCD) = ar (DRC + ar (ACD)

⇒ ar (BCD) = ar(ACD) [∵ ar (DRC) = ar (DPC)]

But, triangles BCD and ACD are on the same base CD.

∴ they are between the same parallels,

i.e., AB || DC

Hence, ABCD is a trapezium. ... (i) **Proved.**

Also, ar (DRC) = ar (DPC) [Given]

Since, triangles DRC and DPC are on the same base CD.

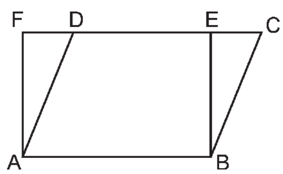
∴ they are between the same parallels,

i.e., DC || RP

Hence, DCPR is a trapezium ... (ii) **Proved.**

EXERCISE 9.4 (Optional)

Q.1. *Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.*



Sol. Given : A parallelogram ABCD and a rectangle ABEF having same base and equal area.

To Prove : 2(AB + BC) > 2(AB + BE)

Proof : Since the parallelogram and the rectangle have same base and equal area, therefore, their attitudes are equal.

Now perimeter of parallelogram ABCD.

= 2 (AB + BC) ... (i)

and perimeter of rectangle ABEF

= 2 (AB + BE) ... (ii)

In ΔBEC, ∠BEC = 90°

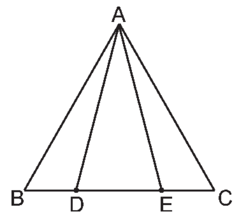
∴ ∠BCE is an acute angle.

∴ BE < BC ... (iii) [Side opposite to smaller angle is smaller]

∴ From (i), (ii) and (iii) we have

2(AB + BC) > 2(AB + BE) **Proved.**

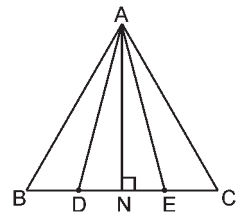
Q.2. *In figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).*



Can you now answer the question that you have left the 'Introduction' of this chapter, whether the field of Budhia *has been actually divided into three parts of equal area?*

[Remark : Note that by taking BD = DE = CE, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n *equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]*

Sol. Given : A triangle ABC, in which D and E are the two points on BC such that BD = DE = EC



To Prove : ar (ABD) = ar (ADE) = ar (AEC)

Construction : Draw AN ⊥ BC

Now, ar (ABD) = $\frac{1}{2}$ × base × altitude (of ΔABD)

= $\frac{1}{2}$ × BD × AN

= $\frac{1}{2}$ × DE × AN [As BD = DE]

= $\frac{1}{2}$ × base × altitude (of ΔADE)

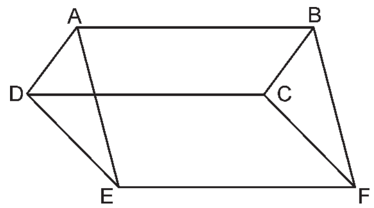
= ar (ADE)

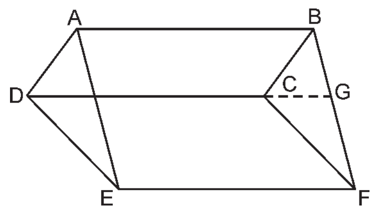
Similarly, we can prove that

ar (ADE) = ar (AEC)

Hence, ar (ABD) = ar (ADE) = ar (AEC) **Proved.**

Q.3. *In the figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar(BCF)*





Sol. Given : Three parallelograms ABCD, DCFE and ABFE.

To Prove : ar (ADE) = ar (BCF)

Construction : Produce DC to intersect BF at G.

Proof : ∠ADC = ∠BCG ... (i) [Corresponding angles]

∠EDC = ∠FCG ... (ii) [Corresponding angles]

⇒ ∠ADC + ∠EDC = ∠BCG + ∠FCG [By adding (i) and (ii)]

⇒ ∠ADE = ∠BCF ... (iii)

In ΔADE and ΔBCF, we have

AD = BC [Opposite sides of || gm ABCD]

DE = CF [Opposite sides of || gm DCEF]

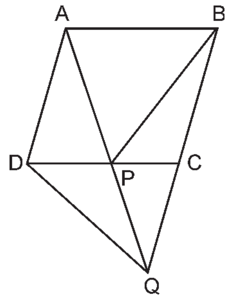
$\angle ADE = \angle BCF$ [From (iii)]

$\therefore \Delta ADE \cong \Delta BCF$ [SAS congruence]

$\Rightarrow$ ar (ADE) = ar (BCF)

[Congruent triangles are equal in area] **Proved.**

Q.4. *In the figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).*



Sol. Given : ABCD is a parallelogram, in which BC is produced to a point Q such that AD = CQ and AQ intersects DC at P.

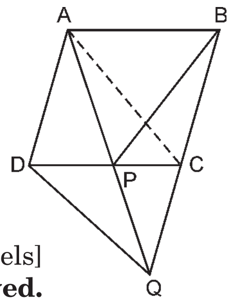
To Prove : ar(BPC) = ar (DPQ)

Construction : Join AC.

Proof : Since AD || BC $\Rightarrow$ AD || BQ

ar (ΔADC) = ar (ΔADQ) ... (i)

[Having same base AD and between same parallel]



$\Rightarrow$ ar (ΔADP) + ar (ΔAPC) = ar (ΔADP) + ar (ΔDPQ)

[From figure]

$\Rightarrow$ ar (ΔAPC) = ar (ΔDPQ)

Now, since AB || DC $\Rightarrow$ AB || PC

ar (ΔAPC) = ar (ΔBPC) ... (ii)

[Having same base PC and between same parallels]

$\Rightarrow$ ar (ΔBPC, = ar (ΔDPQ) [From (i) and (ii)] **Proved.**

Q.5. *In figugre, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. IF AE intersects BC at F, show that*

(i) ar (BDE) = $\frac{1}{4}$ *ar(ABC)*

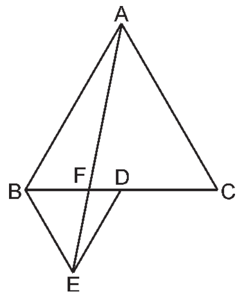
(ii) ar (BDE) = $\frac{1}{2}$ *ar(ABC)*

(iii) ar (ABC) = 2 ar(BEC)

(iv) ar (BEF = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ *ar (AFC)*



[Hint : Join EC and AD. Show that BE | | AC and DE | | AB, etc.]

Sol. Given : ABC and BDE are equilateral triangles, D is the mid-point of BC and AE intersects BC at F.

Construction : Join AD abd EC.

Proof : $\angle ACB = 60°$

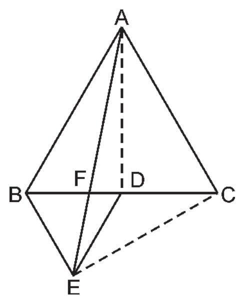
[Angle of an equilateral triangle] ..(1)

$\angle EBC = 60°$

$\Rightarrow \angle ACB = \angle EBC$ [Same reason]

$\Rightarrow$ AC | | BE [Alternate angle are equal]

Similarly, we can prove that AB| |DE ... (2)



(i) D is the mid-point of BC, so AD is a median of ΔABC

$\therefore$ ar (ABD) = $\frac{1}{2}$ ar (ABC) ... (3)

ar(DEB) = ar(DEA) [Triangles on the same base DE and between the same parallels DE and AB]

$\Rightarrow$ ar(DEB) = ar(ADF) + ar(DEF) ...(4)

Also, ar(DEB) = $\frac{1}{2}$ ar(BEC) [DE is a median]

$\Rightarrow$ = $\frac{1}{2}$ ar(BEA) [Triangles on the same base DE and between the same parallels BE and AC] ... (5)

$\Rightarrow$ 2 ar(DEB) = ar(BEA)

$\Rightarrow$ 2 ar(DEB) = ar(ABF) + ar(BEF) ... (6)

Adding (4) and (6), we get

3 ar(DEB) = ar(ADF) + ar(DEF) + ar(ABF) + ar(BEF)

$\Rightarrow$ 3 ar(DEB) = ar(ADF) + ar(ABF) + ar(DEF) + ar(BEF)

= ar(ABD) + ar(BDE)

$\Rightarrow$ 2 ar(DEB) = ar(ABD)

$\Rightarrow$ ar(DEB) = $\frac{1}{2}$ ar(ABC) [From (3)]

$\Rightarrow$ ar(DEB) = $\frac{1}{4}$ ar(ABC) **Proved.**

(ii) From (5) above, we have

ar(BDE) = $\frac{1}{2}$ ar(BAE) **Proved.**

(iii) ar(DEB) = $\frac{1}{2}$ ar(BEC) [DE is a median]

$\Rightarrow$ $\frac{1}{4}$ ar(ABC) = ar $\frac{1}{2}$ (BEC) [From part (i)]

$\Rightarrow$ ar(ABC) = 2 ar (BEC) **Proved.**

(iv) ar(DEB) = ar(BEA) [Triangles on the same base DE and between the same parallels DE AB] ... (7)

$\Rightarrow$ ar(DEB) – ar(DEF) = ar (DEA) – ar (DEF)

$\Rightarrow$ ar(BFE) = ar(AFD) **Proved.**

(v) *****

(vi) From (v), we have ar(FED) $= \frac{1}{2}$ ar(BFE)

$= \frac{1}{2}$ ar(AFD) [From part (iv)]

Now ar (AFC) = ar(AFD) + ar(ADC)

= ar(AFD) + $\frac{1}{2}$ ar(ABC) [BE is a median]

= ar(AFD) + 2ar(BDE) [From part (i)]

= ar(AFD) + 2ar(ADE)

= ar(AFD) + 2ar(AFD) + 2 ar(DEF)

= 3 ar(AFD) + ar(BFE) [From part (v)]

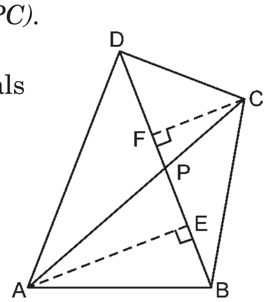
= 3 ar(AFD) + ar(AFD) [From part (iv)]

= 4 ar(AFD)

$\therefore$ $\frac{1}{8}$ ar(AFC) $= \frac{1}{2}$ ar (AFD)

= ar (FED) (From above] **Proved.**

Q.6. *Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar(APB) × ar(CPD) = ar (APD) × ar (BPC). Hint : From A and C, draw perpendiculars to BD.]*



Sol. Given : AB CD is a quadrilateral whose diagonals intersect each other at P.

Construction : Draw AE ⊥ BD and CF ⊥ BD.

Proof : ar (APB) $= \frac{1}{2} \times PB \times AE$ ... (i)

ar (CPD) $= \frac{1}{2} \times DP \times CF$ ... (ii)

Now, ar (BPC) $= \frac{1}{2} \times BP \times CF$ ... (iii)

ar (APD) $= \frac{1}{2} \times DP \times AE$ ... (iv)

From (i) and (ii),

ar (APB) × ar (CPD) $= \frac{1}{4} \times PB \times DP \times AE \times CF$ ... (v)

From (iii) and (iv), we have

ar (BPC) × ar (APD) $= \frac{1}{4} \times BP \times DP \times CF \times AE$ ... (vi)

$\therefore$ ar (APB) × ar (CPD) = ar (BPC) × ar (APD)

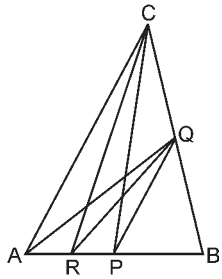
[From (v) and (vi)] **proved**

Q.7. *P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that*

(i) ar (PQR) $= \frac{1}{2}$ ar(ARC) *(ii) ar (RQC) $= \frac{3}{8}$ ar (ABC)*

(iii) ar(PBQ) = ar(ARC)

Sol. **Given :** A triangle ABC, P and Q are mid-points of AB and BC, R is the mid point of AP.



Proof : CP is a median of ΔABC

$\Rightarrow$ ar (APC) = ar (PBC) = ar $\frac{1}{2}$ (ABC)

median divides a triangle into two triangles of equal area] ... (1)

CR is a median of ΔAPC

$\therefore$ ar(ARC) = ar(PRC) = $\frac{1}{2}$ ar(APC) ...(2)

QR is a median of ΔAPQ.

$\therefore$ ar(ARQ) = ar(PRQ) = $\frac{1}{2}$ ar(APQ) ...(3)

PQ is a median of ΔPBC

$\therefore$ ar(PQC) = ar(PQB) = $\frac{1}{2}$ ar(PBC) ...(4)

PQ is a median of ΔRBC

ar(RQC) = ar(PQC) = $\frac{1}{2}$ ar(RBC) ...(5)

(i) ar(PQA) = ar (PQC) [Triangles on the same base PQ and between the same parallels PQ and AC]

$\Rightarrow$ ar(ARQ) + ar(PQR) = $\frac{1}{2}$ ar (PBC) [From (4)]

$\Rightarrow$ ar(PRQ) + ar(PRQ) = $\frac{1}{2}$ ar (APC) [From (3) and (1)]

$\Rightarrow$ 2 ar(PRQ) = ar(ARC) [From (2)]

$\Rightarrow$ ar(PRQ) = $\frac{1}{2}$ ar(ARC) **Proved.**

(ii) From (5), we have

ar(RQC) = $\frac{1}{2}$ ar (RBC)

= $\frac{1}{2}$ ar(PBC) + $\frac{1}{2}$ ar(PRC)

= $\frac{1}{4}$ ar(ABC) + $\frac{1}{4}$ ar(APC) [From (1) and (2)]

= $\frac{1}{4}$ ar(ABC) + $\frac{1}{8}$ ar(ABC) [From (1)]

= ar(RQC) = $\frac{3}{8}$ ar(ABC) **Proved.**

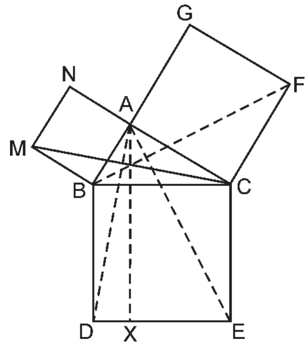
(iii) ar(PBQ) = $\frac{1}{2}$ ar(PBC) [From (4)]

= $\frac{1}{4}$ ar(ABC) [From (1)] ... (6)

ar(ARC) = $\frac{1}{2}$ ar(APC) [From (2)]

$= \frac{1}{4}$ ar(ABC)   [From (1)]   ... (7)

From (6) and (7) we have ar(PBQ) = ar (ARC) **Proved.**



Q.8. *In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that :*

(i) ΔMBC ≅ ΔABD

(ii) ar(BYXD) = 2 ar (MBC)

(iii) ar(BYXD) = ar (ABMN)

(iv) ΔFCB ≅ ΔACE

(v) ar(CYXE) = 2 ar (FCB)

(vi) ar(CYXE) = ar (ACFG)

(vii) ar(BCED)) = ar (ADMN + ar (ACFG)

***Note :** Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in Class X.*

Sol. (i) In DMBC and ΔABD, we have

MB = AB   [Sides of a square]

BC = BD   [Sides of a square]

∠MBC = ∠ABD   [∠MBC = 90° + ∠ABC, and ∠ABC = 90° + ∠ABC]

∴ ΔMBC ≅ ΔABD   [SAS congruent]

(ii) ar(ΔMBC) ≅ ar(ABD)   [Congruent triangles have equal area]

$\Rightarrow \frac{1}{2} \times BC \times \text{height} = \frac{1}{2} \times BD \times BY$

⇒ Height of ΔMBC = BY   [BC = BD]

$\therefore \text{ar(MBC)} = \frac{1}{2} \times BD \times BY$

⇒ Height of ΔMBC = BY   [BC = BD]

$\therefore \text{ar(MBC)} = \frac{1}{2} \times BC \times BY$

⇒ 2 ar(MBC) = BC × BY   ... (1)

Also, ar(BY × D) = BD × BY

= BC × BY   [BC = BD]   ... (2)

From (1) and (2), we have ar(BY × D) = ar (MBC) **Proved.**

(iii) ar(BY × D) = 2·ar(MBC)   [From part (ii)]

$= 2 \times \frac{1}{2} \times$ MB × height of MBC corresponding to BC

= MB × AB   [MB||NC and AB ⊥ MB]

= AB × AB   [∵ AB = MB]

= AB^2

⇒ ar(BY × D) = ar (ABMN) **Proved.**

(iv) In ΔFCB and ΔACR, we have

$FC = AC$ [Sides of a square]

$BC = CE$ [Sides of a square]

$\angle FCB \cong \Delta ACE$ [SAS congruence] **Proved.**

(v) $\frac{1}{2} \times BC \times \text{height} = \frac{1}{2} \times CE \times CY$

$\Rightarrow$ Height of $\Delta FCB = CY$ [BC = CE]

$\therefore$ $ar[FCB] = \frac{1}{2} \times BC \times CY$

$\Rightarrow 2ar[FCB] = BC \times CY$ ... (3)

Also, $ar(CYXE) = CE \times CY$

$= BC \times CY$ ... (4)

From (3) and (4), we have

$ar(CYXE) = 2\ ar(FCB)$ **Proved.**

(vi) $ar(CYXE) = 2 \times \frac{1}{2} \times FC \times$ height of ΔFCB corresponding to FC

$= FC \times AC$ [FC || GB and $AC \perp FC$]

$= AC \times AC$ [AC = FC]

$= AC^2$

$\Rightarrow 2ar(CYXE) = ar(ACFG)$ **Proved.**

(vii) From (iii) and (vi), we have

$ar(BYXD) + ar(CYXE) = ar(ABMN) + ar(ACFG)$

$\Rightarrow ar(BCED) + ar(ABMN) + ar(ACFG)$ **Proved.**