10 CIRCLES

EXERCISE 10.1

Q.1. *Fill in the blanks :*

(i) The centre of a circle lies in __________ of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in _________ of the circle. (exterior/interior)

(iii) The longest chord of a circle is a _________ of the circle.

(iv) An arc is a _________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and _________ of the circle.

(vi) A circle divides the plane, on which it lies in _________ parts.

Sol. (i) interior (ii) exterior (iii) diameter (iv) semicircle (v) the chord (vi) three

Q.2. *Write True or False: Give reasons for your answers.*

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

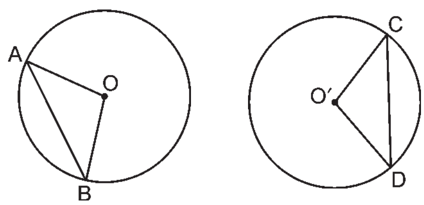
(vi) A circle is a plane figure.

Sol. (i) True (ii) False (iii) False (iv) True (v) False (vi) True

EXERCISE 10.2

Q.1. *Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.*

Sol. Given : Two congruent circles with centres O and O′. AB and CD are equal chords of the circles with centres O and O′ respectively.



To Prove : $\angle AOB = \angle COD$

Proof : In triangles AOB and COD,

$AB = CD$ [Given]

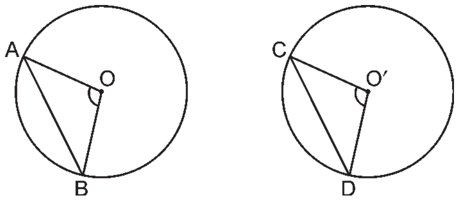
$\left.\begin{array}{l} AO = CO' \\ BO = DO' \end{array}\right\}$ [Radii of congruent circle]

$\Rightarrow$ $\Delta AOB \cong \Delta CO'D$ [SSS axiom]

$\Rightarrow$ $\angle AOB \cong \angle CO'D$ **Proved.** [CPCT]

Q.2. *Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.*

Ans. **Given :** Two congruent circles with centres O and O′. AB and CD are chords of circles with centre O and O′ respectively such that ∠AOB = ∠CO′D



To Prove : AB = CD

Proof : In triangles AOB and CO′D,

AO = CO′ }
BO = DO′ } [Radii of congruent circle]

∠AOB = ∠CO′D [Given]

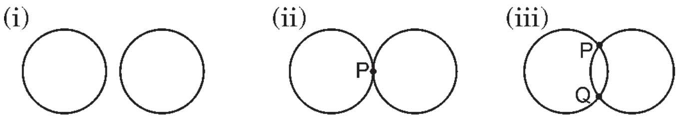
⇒ ΔAOB ≅ ΔCO′D [SAS axiom]

⇒ AB = CD **Proved.** [CPCT]

EXERCISE 10.3

Q.1. *Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?*

Ans.

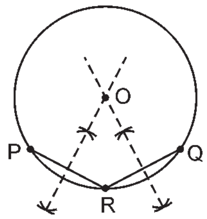


(i) 0 point (ii) 1 point (iii) 2 points

Maximum number of common points = 2 **Ans.**

Q.2. *Suppose you are given a circle. Give a construction to find its centre.*

Ans. **Steps of Construction :**



1. Take arc PQ of the given circle.
2. Take a point R on the arc PQ and draw chords PR and RQ.
3. Draw perpendicular bisectors of PR and RQ. These perpendicular bisectors intersect at point O.

Hence, point O is the centre of the given circle.

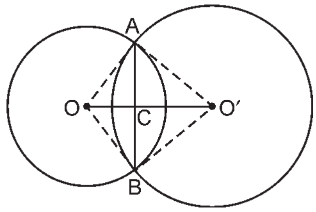
Q.3. *If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.*

Ans. **Given :** AB is the common chord of two intersecting circles (O, r) and (O', r').

To Prove : Centres of both circles lie on the perpendicular bisector of chord AB, i.e., AB is bisected at right angle by OO′.

Construction : Join AO, BO, AO′ and BO′.

Proof : In ΔAOO′ and ΔBOO′



AO = OB (Radii of the circle (O, r)

AO′ = BO′ (Radii of the circle (O′, r'))

OO′ = OO′ (Common)

∴ ΔAOO′ ≅ ΔBOO′ (SSS congruency)

⇒ ∠AOO′ = ∠BOO′ (CPCT)

Now in ΔAOC and ΔBOC

∠AOC = ∠BOC (∠AOO′ = ∠BOO′)

AO = BO (Radii of the circle (O, r))

OC = OC (Common)

$\therefore \quad \Delta AOC \cong \Delta BOC$ (SAS congruency)

$\Rightarrow \quad AC = BC$ and $\angle ACO = \angle BCO$ ...(i) (CPCT)

$\Rightarrow \angle ACO + \angle BCO = 180°$ ..(ii) (Linear pair)

$\Rightarrow \angle ACO = \angle BCO = 90°$ (From (i) and (ii))

Hence, OO′ lie on the perpendicular bisector of AB. **Proved.**

EXERCISE 10.4

Q.1. *Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.*

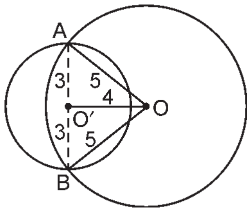
Sol. In $\Delta AOO'$,

$AO^2 = 5^2 = 25$

$AO'^2 = 3^2 = 9$

$OO'^2 = 4^2 = 16$

$AO'^2 + OO'^2 = 9 + 16 = 25 = AO^2$



$\Rightarrow \quad \angle AO'O = 90°$

[By converse of pythagoras theorem]

Similarly, $\angle BO'O = 90°$.

$\Rightarrow \quad \angle AO'B = 90° + 90° = 180°$

$\Rightarrow \quad AO'B$ is a straight line. whose mid-point is O.

$\Rightarrow AB = (3 + 3)$ cm $= 6$ cm **Ans.**

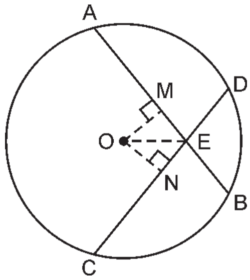
Q.2. *If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.*

Sol. **Given :** AB and CD are two equal chords of a circle which meet at E.

To prove : AE = CE and BE = DE

Construction : Draw OM ⊥ AB and ON ⊥ CD and join OE.

Proof : In ΔOME and ΔONE



OM = ON [Equal chords are equidistant]

OE = OE [Common]

$\angle OME = \angle ONE$ [Each equal to 90°]

$\therefore \quad \Delta OME \cong \Delta ONE$ [RHS axiom]

$\Rightarrow \quad EM = EN$ ...(i) [CPCT]

Now AB = CD [Given]

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow \quad AM = CN$ ..(ii) [Perpendicular from centre bisects the chord]

Adding (i) and (ii), we get

EM + AM = EN + CN

$\Rightarrow \quad AE = CE$ ..(iii)

Now, AB = CD ..(iv)

$\Rightarrow AB - AE = CD - AE$ [From (iii)]

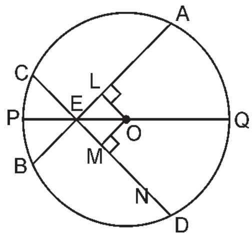
$\Rightarrow \quad BE = CD - CE$ **Proved.**

Q.3. *If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.*

Sol. **Given :** AB and CD are two equal chords of a circle which meet at E within the circle and a line PQ joining the point of intersection to the centre.

To Prove : $\angle AEQ = \angle DEQ$

Construction : Draw OL ⊥ AB and OM ⊥ CD.



Proof : In ΔOLE and ΔOME, we have

OL = OM [Equal chords are equidistant]

OE = OE [Common]

∠OLE = ∠OME [Each = 90°]

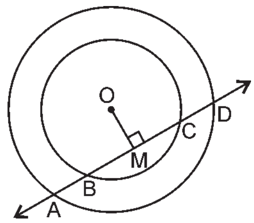
∴ ΔOLE ≅ ΔOME [RHS congruence]

⇒ ∠LEO = ∠MEO [CPCT]

Q.4. *If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig.)*

Sol. Given : A line AD intersects two concentric circles at A, B, C and D, where O is the centre of these circles.

To prove : AB = CD



Construction : Draw OM ⊥ AD.

Proof : AD is the chord of larger circle.

∴ AM = DM ..(i) [OM bisects the chord]

BC is the chord of smaller circle

∴ BM = CM ..(ii) [OM bisects the chord]

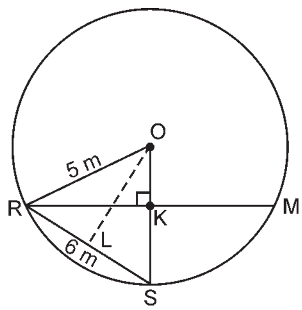
Subtracting (ii) from (i), we get

AM – BM = DM – CM

⇒ AB = CD **Proved.**

Q.5. *Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius* 5 *m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is* 6 *m each, what is the distance between Reshma and Mandip?*

Sol. Let Reshma, Salma and Mandip be represented by R, S and M respectively.



Draw OL ⊥ RS,

$OL^2 = OR^2 - RL^2$

$OL^2 = 5^2 - 3^2$ [RL = 3 m, because OL ⊥ RS]

$= 25 - 9 = 16$

$OL = \sqrt{16} = 4$

Now, area of triangle ORS $= \frac{1}{2} \times KR \times 05$

$= \frac{1}{2} \times KR \times 05$

Also, area of ΔORS $= \frac{1}{2} \times RS \times OL = \frac{1}{2} \times 6 \times 4 = 12 \text{ m}^2$

$\Rightarrow \frac{1}{2} \times KR \times 5 = 12$

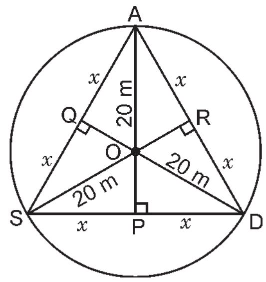
$\Rightarrow KR = \frac{12 \times 2}{5} = \frac{24}{5} = 4.8 \text{ m}$

⇒ RM = 2KR

⇒ RM = 2 × 4.8 = 9.6 m

Hence, distance between Reshma and Mandip is 9.6 m **Ans.**

Q.6. *A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are siting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.*



Sol. Let Ankur, Syed and David be represented by A, S and D respectively.

Let PD = SP = SQ = QA = AR = RD = x

In ΔOPD,

$$OP^2 = 400 - x^2$$

$$\Rightarrow \quad OP = \sqrt{400 - x^2}$$

$$\Rightarrow \quad AP = 2\sqrt{400 - x^2} + \sqrt{400 - x^2}$$

[∵ centroid divides the median in the ratio 2 : 1]

$$= 3\sqrt{400 - x^2}$$

Now, in ΔAPD,

$PD^2 = AD^2 - DP^2$

$$\Rightarrow \quad x^2 = (2x)^2 - \left(3\sqrt{400 - x^2}\right)^2$$

$$\Rightarrow \quad x^2 = 4x^2 - 9(400 - x^2)$$

$$\Rightarrow \quad x^2 = 4x^2 - 3600 + 9x^2$$

$$\Rightarrow \quad 12x^2 = 3600$$

$$\Rightarrow \quad x^2 = \frac{3600}{12} = 300$$

$$\Rightarrow \quad x = 10\sqrt{3}$$

Now, SD = $2x$ = 2 × $10\sqrt{3}$ = $20\sqrt{3}$

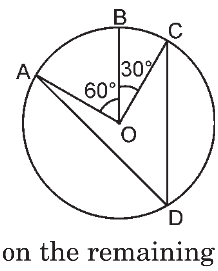
∴ ASD is an equilateral triangle.

⇒ SD = AS = AD = $20\sqrt{3}$

Hence, length of the string of each phone is $20\sqrt{3}$ m **Ans.**

EXERCISE 10.5

Q.1. *In the figure, A, B and C are three points on a circle with centre O such that ∠ BOC = 30° and ∠ AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ ADC.*



Sol. We have, ∠BOC = 30° and ∠AOB = 60°

∠AOC = ∠AOB + ∠BOC = 60° + 30° = 90°

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

∴ 2∠ADC = ∠AOC

$$\Rightarrow \quad \angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 90^\circ \quad \Rightarrow \angle ADC = 45^\circ$$ **Ans.**

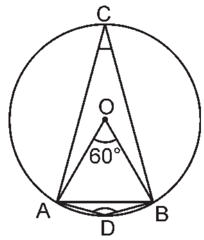
Q.2. *A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.*

Sol. We have, OA = OB = AB

Therefore, ΔOAB is a equilateral triangle.

⇒ $\angle AOB = 60°$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.



∴ $\angle AOB = 2\angle ACB$

⇒ $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60°$

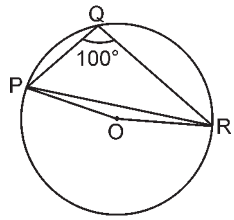
⇒ $\angle ACB = 30°$

Also, $\angle ADB = \frac{1}{2}$ reflex $\angle AOB$

$= \frac{1}{2}(360° - 60°) = \frac{1}{2} \times 300° = 150°$

Hence, angle subtended by the chord at a point on the minor arc is 150° and at a point on the major arc is 30° **Ans.**

Q.3. *In the figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.*



Sol. Reflex angle POR $= 2\angle PQR$

$= 2 \times 100° = 200°$

Now, angle POR $= 360° - 200 = 160°$

Also, PO = OR [Radii of a circle]

∴ $\angle OPR = \angle ORP$

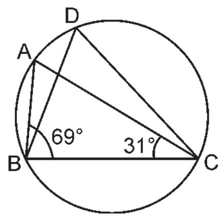
[Opposite angles of isosceles triangle]

In ΔOPR, $\angle POR = 160°$

∴ $\angle OPR = \angle ORP = 10°$

[Angle sum property of a triangle]. **Ans.**

Q.4. *In the figure, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.*



Sol. In ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180°$

[Angle sum property of a triangle]

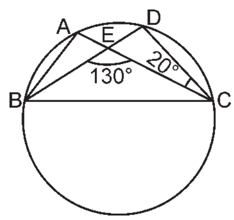
⇒ $69° + 31° + \angle BAC = 180°$

⇒ $\angle BAC = 180° - 100° = 80°$

Also, $\angle BAC = \angle BDC$ [Angles in the same segment]

⇒ $\angle BDC = 80°$ **Ans.**

Q.5. *In the figrue, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.*



Sol. $\angle BEC + \angle DEC = 180°$ [Linear pair]

⇒ $130° + \angle DEC = 180°$

⇒ $\angle DEC = 180° - 130° = 50°$

Now, in ΔDEC,

⇒ $\angle DEC + \angle DCE + \angle CDE = 180°$

[Angle sum property of a triangle]

⇒ $50° + 20° + \angle CDE = 180°$

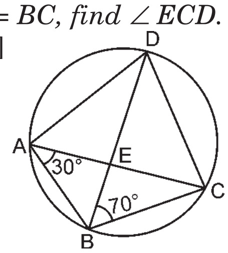
⇒ $\angle CDE = 180° - 70° = 110°$

Also, $\angle CDE = \angle BAC$ [Angles in same segment]

⇒ $\angle BAC = 110°$ **Ans.**

Q.6. *ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC = 30°$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.*

Sol. $\angle CAD = \angle DBC = 70°$ [Angles in the same segment]



Therefore, $\angle DAB = \angle CAD + \angle BAC$

$= 70° + 30° = 100°$

But, $\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral]

So, $\angle BCD = 180° - 100° = 80°$

Now, we have AB = BC

Therefore, $\angle BCA = 30°$ [Opposite angles of an isosceles triangle]

Again, $\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral]

$\Rightarrow 100° + \angle BCA + \angle ECD = 180°$ [$\because \angle BCD = \angle BCA + \angle ECD$]

$\Rightarrow 100° + 30° + \angle ECD = 180°$

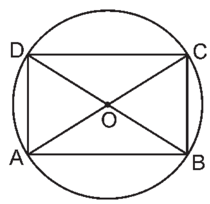
$\Rightarrow 130° + \angle ECD = 180°$

$\Rightarrow \angle ECD = 180° - 130° = 50°$

Hence, $\angle BCD = 80°$ and $\angle ECD = 50°$ **Ans.**

Q.7. *If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.*

Sol. Given : ABCD is a cyclic quadrilateral, whose diagonals AC and BD are diameter of the circle passing through A, B, C and D.



To Prove : ABCD is a rectangle.

Proof : In ΔAOD and ΔCOB

AO = CO [Radii of a circle]

OD = OB [Radii of a circle]

$\angle AOD = \angle COB$ [Vertically opposite angles]

$\therefore$ $\Delta AOD \cong \Delta COB$ [SAS axiom]

$\therefore$ $\angle OAD = \angle OCB$ [CPCT]

But these are alternate interior angles made by the transversal AC, intersecting AD and BC.

$\therefore$ AD || BC

Similarly, AB || CD.

Hence, quadrilateral ABCD is a parallelogram.

Also, $\angle ABC = \angle ADC$ ..(i) [Opposite angles of a ||gm are equal]

And, $\angle ABC + \angle ADC = 180°$ ...(ii)

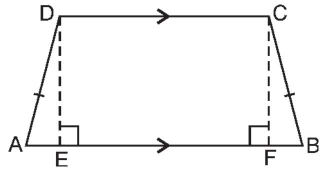
[Sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow \angle ABC = \angle ADC = 90°$ [From (i) and (ii)]

$\therefore$ ABCD is a rectangle. [A ||gm one of whose angles is 90° is a rectangle] **Proved.**

Q.8. *If the non-parallel sides of a trapezium are equal, prove that it is cyclic.*

Sol. Given : A trapezium ABCD in which AB || CD and AD = BC.



To Prove : ABCD is a cyclic trapezium.

Construction : Draw DE ⊥ AB and CF ⊥ AB.

Proof : In ΔDEA and ΔCFB, we have

AD = BC [Given]

$\angle DEA = \angle CFB = 90°$ [DE ⊥ AB and CF ⊥ AB]

DE = CF

[Distance between parallel lines remains constant]

∴ ΔDEA ≅ ΔCFB [RHS axiom]

⇒ ∠A = ∠B ...(i) [CPCT]

and, ∠ADE = ∠BCF ..(ii) [CPCT]

Since, ∠ADE = ∠BCF [From (ii)]

⇒ ∠ADE + 90° = ∠BCF + 90°

⇒ ∠ADE + ∠CDE = ∠BCF + ∠DCF

⇒ ∠D = ∠C ..(iii)

[∠ADE + ∠CDE = ∠D, ∠BCF + ∠DCF = ∠C]

∴ ∠A = ∠B and ∠C = ∠D [From (i) and (iii)] (iv)

∠A + ∠B + ∠C + ∠D = 360° [Sum of the angles of a quadrilateral is 360°]

⇒ 2(∠B + ∠D) = 360° [Using (iv)]

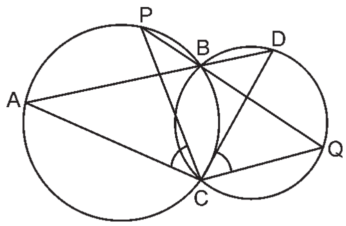
⇒ ∠B + ∠D = 180°

⇒ Sum of a pair of opposite angles of quadrilateral ABCD is 180°.

⇒ ABCD is a cyclic trapezium **Proved.**

Q.9. *Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that ∠ ACP = ∠QCD.*

Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.



To Prove : ∠ACP = ∠QCD.

Proof : ∠ACP = ∠ABP ...(i)

[Angles in the same segment]

∠QCD = ∠QBD ..(ii)

[Angles in the same segment]

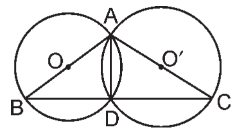
But, ∠ABP = ∠QBD ..(iii) [Vertically opposite angles]

By (i), (ii) and (ii) we get

∠ACP = ∠QCD **Proved.**

Q.10. *If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.*

Sol. Given : Sides AB and AC of a triangle ABC are diameters of two circles which intersect at D.



To Prove : D lies on BC.

Proof : Join AD

∠ADB = 90° ...(i) [Angle in a semicircle]

Also, ∠ADC = 90° ..(ii)

Adding (i) and (ii), we get

∠ADB + ∠ADC = 90° + 90°

⇒ ∠ADB + ∠ADC = 180°

⇒ BDC is a straight line.

∴ D lies on BC

Hence, point of intersection of circles lie on the third side BC. **Proved.**

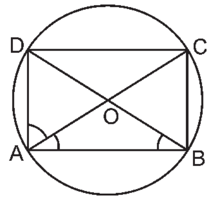
Q.11. *ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.*

Sol. Given : ABC and ADC are two right triangles with common hypotenuse AC.

To Prove : ∠CAD = ∠CBD

Proof : Let O be the mid-point of AC.

Then OA = OB = OC = OD



Mid point of the hypotenuse of a right triangle is equidistant from its vertices with O as centre and radius equal to OA, draw a circle to pass through A, B, C and D.

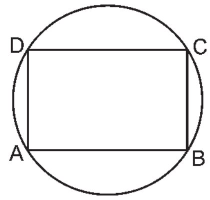
We know that angles in the same segment of a circle are equal.

Since, ∠CAD and ∠CBD are angles of the same segment.

Therefore, ∠CAD = ∠CBD. **Proved.**

Q.12. *Prove that a cyclic parallelogram is a rectangle.*

Sol. **Given :** ABCD is a cyclic parallelogram.



To prove : ABCD is a rectangle.

Proof : ∠ABC = ∠ADC ...(i)

[Opposite angles of a ||gm are equal]

But, ∠ABC + ∠ADC = 180° ... (ii)

[Sum of opposite angles of a cyclic quadrilateral is 180°]

⇒ ∠ABC = ∠ADC = 90° [From (i) and (ii)]

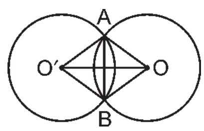
∴ ABCD is a rectangle

[A ||gm one of whose angles is 90° is a rectangle]

Hence, a cyclic parallelogram is a rectangle. **Proved.**

EXERCISE 10.6 (Optional)

Q.1. *Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.*



Sol. **Given :** Two intersecting circles, in which OO′ is the line of centres and A and B are two points of intersection.

To prove : ∠OAO′ = ∠OBO′

Construction : Join AO, BO, AO′ and BO′.

Proof : In ΔAOO′ and ΔBOO′, we have

AO = BO [Radii of the same circle]

AO′ = BO′ [Radii of the same circle]

OO′ = OO′ [Common]

∴ ΔAOO′ ≅ ΔBOO′ [SSS axiom]

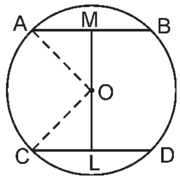
⇒ ∠OAO′ = ∠OBO′ [CPCT]

Hence, the line of centres of two intersecting circles subtends equal angles at the two points of intersection. **Proved.**

Q.2. *Two chords AB and CD of lengths* 5 *cm and* 11 *cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is* 6 *cm, find the radius of the circle.*

Sol. Let O be the centre of the circle and let its radius be *r* cm.

Draw OM ⊥ AB and OL ⊥ CD.



Then, $AM = \frac{1}{2}AB = \frac{5}{2}$ cm

and, $CL = \frac{1}{2}CD = \frac{11}{2}$ cm

Since, AB || CD, it follows that the points O, L, M are

collinear and therefore, LM = 6 cm.
Let OL = x cm. Then OM = $(6 - x)$ cm
Join OA and OC. Then OA = OC = r cm.
Now, from right-angled ΔOMA and ΔOLC, we have
$OA^2 = OM^2 + AM^2$ and $OC^2 = OL^2 + CL^2$ [By Pythagoras Theorem]

$\Rightarrow r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2$ ..(i) and $r^2 = x^2 + \left(\frac{11}{2}\right)^2$ ... (ii)

$\Rightarrow (6 - x)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2$ [From (i) and (ii)]

$\Rightarrow 36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$

$\Rightarrow - 12x = \frac{121}{4} - \frac{25}{4} - 36$

$\Rightarrow - 12x = \frac{96}{4} - 36$

$\Rightarrow - 12x = 24 - 36$

$\Rightarrow - 12x = - 12$

$\Rightarrow \quad x = 1$

Substituting x =1 in (i), we get

$r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2$

$\Rightarrow r^2 = (6 - 1)^2 + \left(\frac{5}{2}\right)^2$

$\Rightarrow r^2 = (5)^2 + \left(\frac{5}{2}\right)^2 = 25 + \frac{25}{4}$

$\Rightarrow r^2 = \frac{125}{4}$

$\Rightarrow r = \frac{5\sqrt{5}}{2}$

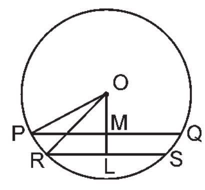
Hence, radius $r = \frac{5\sqrt{5}}{2}$ cm. **Ans.**

Q.3. *The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?*

Sol. Let PQ and RS be two parallel chords of a circle with centre O.
We have, PQ = 8 cm and RS = 6 cm.
Draw perpendicular bisector OL of RS which meets PQ in M. Since, PQ || RS, therefore, OM is also perpendicular bisector of PQ.

Also, OL = 4 cm and RL = $\frac{1}{2}$RS ⇒ RL = 3 cm

and PM = $\frac{1}{2}$PQ ⇒ PM = 4 cm



In ΔORL, we have
$OR^2 = RL^2 + OL^2$ [Pythagoras theorem]

$\Rightarrow OR^2 = 3^2 + 4^2 = 9 + 16$

$\Rightarrow OR^2 = 25 \Rightarrow OR = \sqrt{25}$

$\Rightarrow OR = 5$ cm

$\therefore OR = OP$ [Radii of the circle]

$\Rightarrow OP = 5$ cm

Now, in ΔOPM

$OM^2 = OP^2 - PM^2$ [Pythagoras theorem]

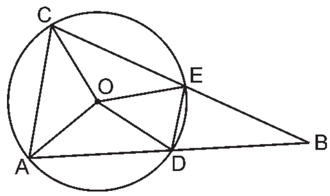
$\Rightarrow OM^2 = 5^2 - 4^2 = 25 - 16 = 9$

$OM = \sqrt{9} = 3$ cm

Hence, the distance of the other chord from the centre is 3 cm. **Ans.**

Q.4. *Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.*

Sol. Given : Two equal chords AD and CE of a circle with centre O. When meet at B when produced.



To Prove : $\angle ABC = \frac{1}{2}(\angle AOC - \angle DOE)$

Proof : Let $\angle AOC = x$, $\angle DOE = y$, $\angle AOD = z$

$\angle EOC = z$ [Equal chords subtends equal angles at the centre]

$\therefore x + y + 2z = 36°$ [Angle at a point] .. (i)

$OA = OD \Rightarrow \angle OAD = \angle ODA$

$\therefore$ In DOAD, we have

$\angle OAD + \angle ODA + z = 180°$

$\Rightarrow 2\angle OAD = 180° - z$ [$\because \angle OAD = \angle OBA$]

$\Rightarrow \angle OAD = 90° - \frac{z}{2}$ ... (ii)

Similarly $\angle OCE = 90° - \frac{z}{2}$ ... (iii)

$\Rightarrow \angle ODB = \angle OAD + \angle ODA$ [Exterior angle property]

$\Rightarrow \angle OEB = 90° - \frac{z}{2} + z$ [From (ii)]

$\Rightarrow \angle ODB = 90° + \frac{z}{2}$ ... (iv)

Also, $\angle OEB = \angle OCE + \angle COE$ [Exterior angle property]

$\Rightarrow \angle OEB = 90° - \frac{z}{2} + z$ [From (iii)]

$\Rightarrow \angle OEB = 90° + \frac{z}{2}$ ... (v)

Also, $\angle OED = \angle ODE = 90° - \frac{y}{2}$ ... (vi)

O from (iv), (v) and (vi), we have

$\angle BDE = \angle BED = 90° + \frac{z}{2} - \left(90° - \frac{y}{2}\right)$

$\Rightarrow \angle BDE = \angle BED = \frac{y+z}{2}$

$\Rightarrow \angle BDE = \angle BED = y + z$ ... (vii)

$\therefore \angle BDE = 180° - (y + z)$

$\Rightarrow \angle ABC = 180° - (y + z)$ ... (viii)

Now, $\frac{y-z}{2} = \frac{360° - y - 2z - y}{2} = 180° - (y + z)$ ... (ix)

From (viii) and (ix), we have

$\angle ABC = \frac{x-y}{2}$ **Proved.**

Q.5. *Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.*

Sol. **Given :** A rhombus ABCD whose diagonals intersect each other at O.

To prove : A circle with AB as diameter passes through O.

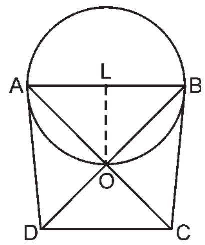
Proof : $\angle AOB = 90°$

[Diagonals of a rhombus bisect each other at 90°]

$\Rightarrow \Delta AOB$ is a right triangle right angled at O.

$\Rightarrow$ AB is the hypotenuse of A B right ΔAOB.

$\Rightarrow$ If we draw a circle with AB as diameter, then it will pass through O. because angle is a semicircle is 90° and $\angle AOB = 90°$ **Proved.**

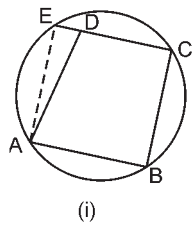


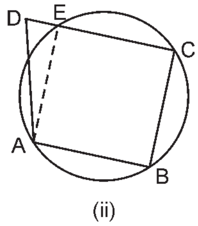
Q.6. *ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.*

Sol. **Given :** ABCD is a parallelogram.

To Prove : AE = AD.

Construction : Draw a circle which passes through ABC and intersect CD (or CD produced) at E.





Proof : For fig (i)

$\angle AED + \angle ABC = 180°$ [Linear pair] ... (ii)

But $\angle ACD = \angle ADC = \angle ABC + \angle ADE$

$\Rightarrow \angle ABC + \angle ADE = 180°$ [From (ii)] ... (iii)

From (i) and (iii)

$\angle AED + \angle ABC = \angle ABC + \angle ADE$

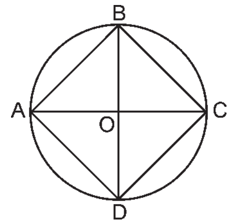
$\Rightarrow \angle AED = \angle ADE$

$\Rightarrow \angle AD = \angle AE$ [Sides opposite to equal angles are equal]

Similarly we can prove for Fig (ii) **Proved.**

Q.7. *AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is rectangle.*

Sol. Given : A circle with chords AB and CD which bisect each other at O.



To Prove : (i) AC and BD are diameters

(ii) ABCD is a rectangle.

Proof : In ΔOAB and ΔOCD, we have

$OA = OC$ [Given]

$OB = OD$ [Given]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\Rightarrow \Delta AOB \cong \angle COD$ [SAS congruence]

$\Rightarrow \angle ABO = \angle CDO$ and $\angle BAO = \angle BCO$ [CPCT]

$\Rightarrow AB \,||\, DC$ ... (i)

Similarly, we can prove $BC \,||\, AD$ ... (ii)

Hence, ABCD is a parallelogram.

But ABCD is a cyclic parallelogram.

$\therefore$ ABCD is a rectangle. [Proved in Q. 12 of Ex. 10.5]

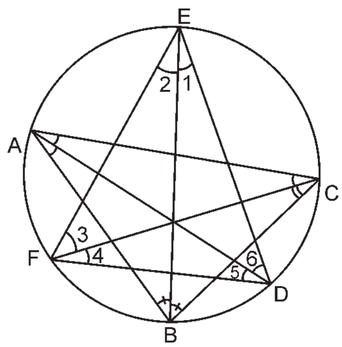
$\Rightarrow \angle ABC = 90°$ and $\angle BCD = 90°$

$\Rightarrow$ AC is a diameter and BD is a diameter

[Angle in a semicircle is 90°] **Proved.**

Q.8. *Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are* $90° - \frac{1}{2}A$, $90° - \frac{1}{2}B$ *and* $90° - \frac{1}{2}C$.

Sol. Given : ΔABC and its circumcircle. AD, BE, CF are bisectors of $\angle A$, $\angle B$, $\angle C$ respectively.



Construction : Join DE, EF and FD.

Proof : We know that angles in the same segment are equal.

$\therefore \quad \angle 5 = \frac{\angle C}{2}$ and $\angle 6 = \frac{\angle B}{2}$ ..(i)

$\angle 1 = \frac{\angle A}{2}$ and $\angle 2 = \frac{\angle C}{2}$ ..(ii)

$\angle 4 = \frac{\angle A}{2}$ and $\angle 3 = \frac{\angle B}{2}$ ..(iii)

From (i), we have

$\angle 5 + \angle 6 = \frac{\angle C}{2} + \frac{\angle B}{2}$

$\Rightarrow \angle D = \frac{\angle C}{2} + \frac{\angle B}{2}$ ...(iv)

[$\because \angle 5 + \angle 6 = \angle D$]

But $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 180° - \angle A$

$\Rightarrow \qquad \frac{\angle B}{2} + \frac{\angle C}{2} = 90° - \frac{\angle A}{2}$

$\therefore$ (iv) becomes,

$\angle D = 90° - \frac{\angle A}{2}$.

Similarly, from (ii) and (iii), we can prove that

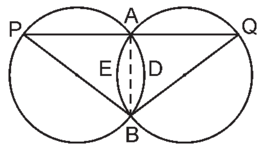
$\angle E = 90° - \frac{\angle B}{2}$ and $\angle F = 90° - \frac{\angle C}{2}$ **Proved.**

Q.9. *Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.*

Sol. **Given :** Two congruent circles which intersect at A and B. PAB is a line through A.

To Prove : BP = BQ.

Construction : Join AB.



Proof : AB is a common chord of both the circles.

But the circles are congruent —

$\Rightarrow$ arc ADB = arc AEB

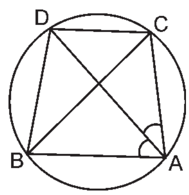
$\Rightarrow \quad \angle APB = \angle AQB$ Angles subtended

$\Rightarrow \quad$ BP = BQ [Sides opposite to equal angles are equal] **Proved.**

Q.10. *In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.*

Sol. Let angle bisector of ∠A intersect circumcircle of ΔABC at D.

Join DC and DB.



$\angle BCD = \angle BAD$

[Angles in the same segment]

$\Rightarrow \angle BCD = \angle BAD \ \frac{1}{2} \ \angle A$

[AD is bisector of ∠A] ...(i)

Similarly $\angle DBC = \angle DAC \ \frac{1}{2} \ \angle A$ ... (ii)

From (i) and (ii) $\angle DBC = \angle BCD$

$\Rightarrow$ BD = DC [sides opposite to equal angles are equal]

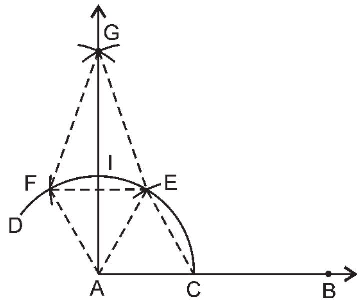
$\Rightarrow$ D lies on the perpendicular bisector of BC.

Hence, angle bisector of ∠A and perpendicular bisector of BC intersect on the circumcircle of ΔABC **Proved.**

11 CONSTRUCTIONS

EXERCISE 11.1

Q.1. *Construct an angle of* 90° *at the initial point of a given ray and justify the construction.*



Steps of Construction

(i) Let us take a ray AB with initial point A.

(ii) Taking A as centre and some radius, draw an arc of a circle, which intersects AB at C.

(iii) With C as centre and the same radius as before, draw an arc, intersecting the previous arc at E.

(iv) With E as centre and the same radius, as before, draw an arc, which intersects the arc drawn in step (ii) at F.

(v) With E as centre and some radius, draw an arc.

(vi) With F as centre and the same radius as before, draw another arc, intersecting the previous arc at G.

(vii) Draw the ray AG.

Then ∠BAG is the required angle of 90°.

Justification : Join AE, CE, EF, FG and GE

AC = CE = AE [By construction]

⇒ ΔACE is an equilateral triangle

⇒ ∠CAE = 60° ... (i)

Similarly, ∠AEF = 60° ... (ii)

From (i) and (ii), FE || AC ... (iii) [Alternate angles are equal]

Also, FG = EG [By construction]

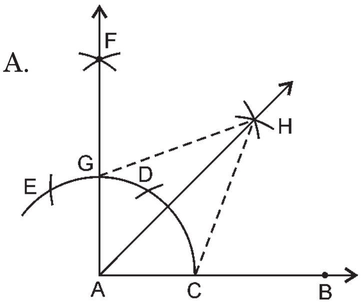
⇒ G lies on the perpendicular bisector of EF

⇒ ∠GIE = 90° ... (iv)

∴ ∠GAB = ∠GIE = 90° [Corresponding angles]

GF = GE [Arcs of equal radii]

Q.2. *Construct an angle of* 45° *at the initial point of a given ray and justify the construction.*



Steps of Construction

(i) Let us take a ray AB with initial point A.

(ii) Draw ∠BAF = 90°, as discussed in Q. 1.

(iii) Taking C as centre and radius more than $\frac{1}{2}$ CG, draw an arc.

(iv) Taking G as centre and the same radius as before, draw another arc, intersecting the previous arc at H.

(v) Draw the ray AH. Then ∠BAH is the required angle of 45°.

Justification : Join GH and CH.

In ΔAHG and ΔAHC, we have

HG = HC [Arcs of equal radii]

AG = AC [Radii of the same arc]

AH = AH [Common]

∴ ΔAHG ≅ ΔAHC [SSS congruence]

⇒ ∠HAG = ∠HAC [CPCT] ... (i)

But ∠HAG + ∠HAC = 90° [By construction] ... (ii)

⇒ ∠HAG = ∠HAC = 45° [From (i) and (ii)]

Q.3. *Construct the angles of the following measurements.*

(i) 30° *(ii)* $22\frac{1}{2}°$ *(iii)* 15°

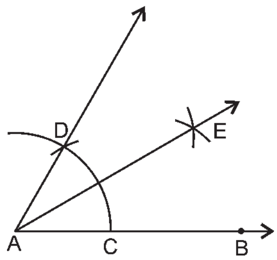
(i) Steps of Construction

(a) Draw a ray AB, with initial point A.

(b) With A as centre and some convenient radius, draw an arc, intersecting AB at C.

(c) With C as centre and the same radius as before, draw another arc, intersecting the previously drawn arc at D.

(d) Draw ray AD.



(e) Now, taking C and D as centres and with the radius more than $\frac{1}{2}$ DC, draw arcs to intersect each other at E.

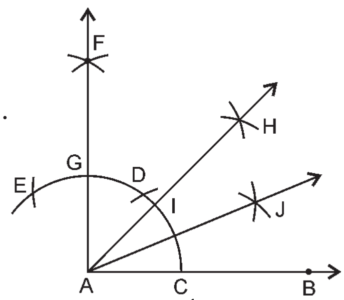
(f) Draw ray AE. Then ∠BAE is the required angle of 30°.

(ii) Steps of Construction

(a) Draw a ray AB with initial point A.

(b) Draw ∠BAH = 45° as discussed in Q. 2.

(c) Taking I and C as centres and with the radius more than $\frac{1}{2}$ CI, draw arcs to intersect each other at J.



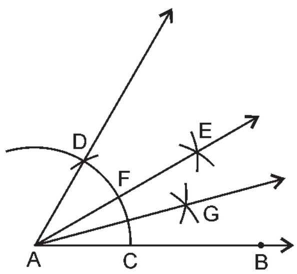
(d) Draw ray AJ. Then ∠BAJ is the required angle of $22\frac{1}{2}°$.

(iii) Steps of Construction

(a) Draw ∠BAE = 30° as discussed in part (i).

(b) Taking C and F as centres and with the radius more than $\frac{1}{2}$ CF, draw arcs to intersect each other at G.

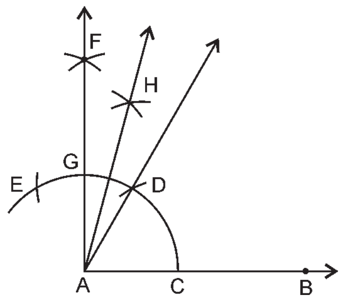
(c) Draw ray AG. Then ∠BAG is the required angle of 15°.



Q.4. *Construct the following angles and verify by measuring them by a protractor.*

(i) 75° *(ii)* 105° *(iii)* 135°

(i) Steps of Construction



(a) Draw a ray AB with initial point A.

(b) With A as centre and any convenient radius, draw an arc, intersecting AB at C.

(c) With C as centre and the same radius, draw an arc, cutting the previous arc at D.

(d) With D as centre and the same radius, draw another arc, cutting the arc drawn in step (b) at E.

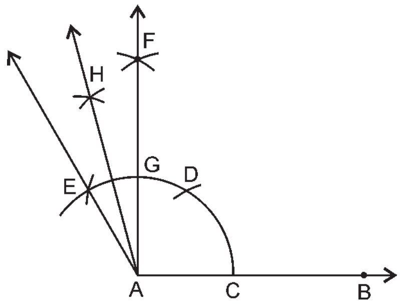
(e) With D and E as centres and some radius, draw arcs to intersect each other at F.

(f) Draw ray AF and AD.

(g) With D and G as centres, and radius more than $\frac{1}{2}$GD, draw arcs to intersect. each other at H.

(h) Draw ray AH. Then $\angle$BAH is the required angle of 75°.
On measuring using a protractor, we find that $\angle$BAH = 75°.

(ii) Steps of Construction



(a) At A, draw an $\angle$BAF = 90°, as discussed in Q. 1.

(b) With A as centre and some convenient radius, draw an arc, intersecting AB at C.

(c) With C as centre and the same radius, draw an arc, which cuts the precious arc at D.

(d) With D as centre and the same radius, draw an arc, which cuts the arc drawn in step (b) at E.

(e) Draw ray AE.

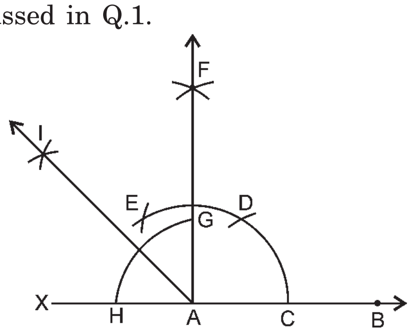
(f) With G and E as centres and radius more than $\frac{1}{2}$GE, draw arcs to intersect each other at H.

(g) Join AH. Then $\angle$BAH is the required angle of 105°.
On measuring using a protractor, we find that $\angle$BAH = 105°.

(iii) Steps of Construction

(a) At A, draw angle BAF = 90°, as discussed in Q.1.

(b) Produce BA to X.

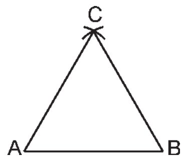


(c) With A as centre and some convenient radius, draw an arc, which cuts AF and AX at G and H respectively.

(d) With G and H as centres and radius more than $\frac{1}{2}$GH, draw arcs to intersect each other at I.

(e) Draw ray AI. Then $\angle$BAI is the required angle of 135°.
On measuring using a protractor, we find that $\angle$BAI = 135°.

Q.5. *Construct an equilateral triangle, given its side and justify the construction.*



(i) Steps of Construction

(i) Draw a line segment AB of given length.

(ii) With A and B as centres and radius equal to AB, draw arcs to intersect each other at C.

(iii) Join AC and BC. Then ABC is the required equilateral triangle.

Justification : AB = AC [By construction]

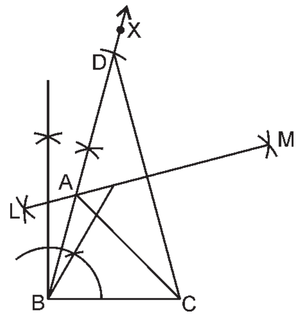
AB = BC [By construction]

⇒ AB = AC = BC

Hence, ΔABC is an equilateral triangle.

EXERCISE 11.2

Q.1. *Construct a triangle ABC in which BC = 7 cm, ∠B = 75° and AB + AC = 13 cm.*



Steps of Construction

(i) Draw a line segment BC = 7 cm.

(ii) At B, draw ∠CBX = 75°.

(iii) Cut a line segment BD = 13 cm from BX.

(iv) Join DC

(v) Draw the perpendicular bisector LM of CD, which intersects BD at A.

(vi) Join AC. Then ABC is the required triangle.

Justification : In ΔACD, we have

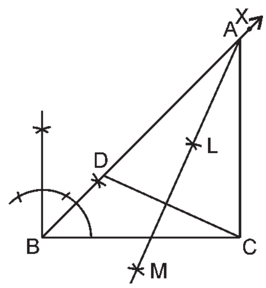
AC = AD [A lies on the perpendicular bisector of DC.]

AB = BD – AD

= BD – AC

⇒ AB + AC = BD

Q.2. *Construct a triangle ABC, in which BC = 8 cm, ∠B = 45° and AB – AC = 3.5 cm.*



Steps of Construction

(i) Draw a line segment BC = 3.5 cm

(ii) At B, draw ∠CBX = 45°.

(iii) From BX, cut off BD = 3.5 cm.

(iv) Join DC.

(v) Draw the perpendicular bisector LM of DC, which intersects BX at A. (vi) Join AC. Then ABC is the required triangle.

Justification : In ΔADC,

AD = AC [A lies on the perpendicular bisector of DC]

BD = AB – AD

⇒ BD = AB – AC

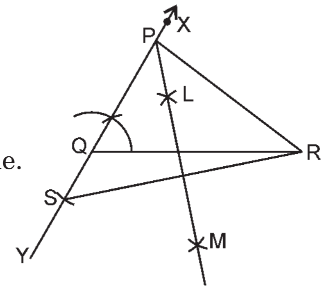
Q.3. *Construct a triangle PQR in which QR = 6 cm, ∠Q = 60° and PR – PQ = 2 cm.*

Steps of Construction

(i) Draw a line segment QR = 6 cm

(ii) At Q, draw ∠RQX = 60°.

(iii) Produce XQ to Y.
(iv) Cut off QS = 2 cm from QY.
(v) Join SR.
(vi) Draw the perpendicular bisector LM of SR, which intersect QX at P.
(vii) Join PR. Then PQR is the required triangle.



Justification : In ΔPSR, we have

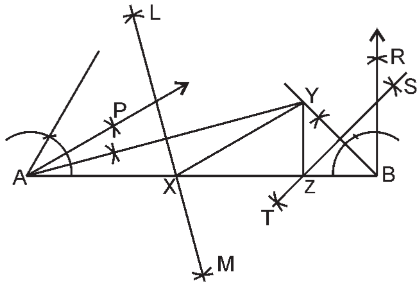
SP = PR [P lies on the perpendicular bisector of SR]

QS = PS − PQ
= PR − PQ

Q.4. *Construct a ΔXYZ in which* $\angle X = 30°$, $\angle Z = 90°$ *and* $XY + YZ + ZX = 11$ *cm.*

Steps of Construction

(i) Draw a line segment AB = 11 cm
(ii) At A, draw ∠BAP = 30° and at B, draw ∠ABR = 90°
(iii) Draw the bisector of ∠BAP and ∠ABR, which intersect each other at Y.
(iv) Join AY and BY.
(v) Draw the perpendicular bisectors LM and ST of AY and BY respectively. LM and ST intersect AB at X and Z respectively.
(vi) Join XY and YZ. Then XYZ is the required triangle.



Justification : In ΔAXY, we have

AX = XY [X lies on the perpendicular bisector of AY] ...(i)

Similarly, ZB = YZ ... (ii)

∴ XY + YZ + ZX = AX + ZB + ZX [From (i) and (ii)]
= AB

From (i), AX = AY

⇒ ∠XAY = ∠XYA [Angles opposite to equal sides are equal] ... (iii)

In ΔAXY, ∠YXZ = ∠XAY + ∠XYA [Exterior angle is equal to sum of interior opposite angles]

⇒ ∠YXZ = 2∠XAY [From (iii)]

⇒ ∠YXZ = ∠XAP [∵ AY bisects ∠XAP]

Similarly, ∠YZX = ∠ZBR.

Q.5. *Construct a right triangle whose base is* 12 *cm and sum of its hypotenuse and other side is* 18 *cm.*

Steps of Construction

(i) Draw a line segment AB = 12 cm.
(ii) At A, draw ∠BAX = 90°.
(iii) From AX, cut off AD = 18 cm.
(iv) Join DB.
(v) Draw the perpendicular bisector LM of BD, which intersects AD at C.
(vi) Join BC. Then ΔABC is the required triangle.